section 1

1a) $f(x) = 2x^3 + x^2 - 7x - 6$

$f(2) = 2 \times 2^3 + 2^2 - 14 - 6$

$= 16 + 4 - 14 - 6$

$= 0$

as $f(2) = 0$, $(x-2)$ is a factor of $f(x)$

b) $(2x^3 + x^2 - 7x - 6) \div (x-2)$

	$2x^2$	$5x$	3
x	$2x^3$	$5x^2$	$3x$
-2	$-4x^2$	$-10x$	-6

$\therefore \; 2x^3 + x^2 - 7x - 6 = (x-2)(2x^2 + 5x + 3)$

$= (x-2)(2x^2 + 2x + 3x + 3)$

$= (x-2)(2x(x+1) + 3(x+1))$

$= (x-2)(2x+3)(x+1)$

$x - 2 = 0$ $\quad$ $2x + 3 = 0$ $\quad$ $x + 1 = 0$

$x = 2$ $\quad$ $2x = -3$ $\quad$ $x = -1$

$x = -\frac{3}{2}$

2) $4^x = 18 - 7(2^x)$

$(2^2)^x = 18 - 7(2^x)$

$(2^x)^2 + 7(2^x) - 18 = 0$

$(2^x + 9)(2^x - 2) = 0$

~~$2^x = 9$~~ $\quad$ $2^x = 2$

$x = 1$

3 $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_2 = 2 = \frac{2}{2}[2a + (2-1)d]$

$2 = 2a + d$

$2a = 2 - d$

$S_{10} = 330 = \frac{10}{2}[2a + (10-1)d]$

$330 = 5(2a + 9d)$

$66 = 2a + 9d$

$2a = 66 - 9d$

a) $\therefore 2 - d = 66 - 9d$

$8d = 64$

$d = 8$

b) $2a = 2 - 8$

$a = -3$

c) $S_n = 1170 = \frac{n}{2}[2 \times -3 + (n-1)8]$

$2340 = n(-6 + 8n - 8)$

$2340 = 8n^2 - 14n$

$8n^2 - 14n - 2340 = 0$

$4n^2 - 7n - 1170 = 0$

$(n-18)(4n+65) = 0$

$\underline{\underline{n = 18}}$ or ~~$n = -\frac{65}{4}$~~

4 $\cos 3x \equiv 4\cos^3 x - 3\cos x$

LHS $\equiv \cos 3x$

$\equiv \cos(2x + x)$

$\equiv \cos 2x \cos x - \sin 2x \sin x$

$\equiv (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$

$\equiv 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$\equiv 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$\equiv 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$\equiv 4\cos^3 x - 3\cos x$

$\equiv$ RHS

$\square$

b) $8\cos^3 x - 6\cos x = \sqrt{3}$

$2(4\cos^3 x - 3\cos x) = \sqrt{3}$

$\cos 3x = \frac{\sqrt{3}}{2}$

$0 \le x \le 2\pi \Rightarrow 0 \le 3x \le 6\pi$

$3x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{35\pi}{6}$

$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

5a) i) $-5\sin x$ ii) $\frac{1}{x}$ iii) $\frac{1}{4}e^x$

b) $y = 2\ln x$ when $x=1$: $y = 2\ln 1 = 0$

$y' = \frac{2}{x}$ $y' = \frac{2}{1} = 2$

$m_1 = 2 \therefore m_2 = -\frac{1}{2}$

$(y - 0) = -\frac{1}{2}(x-1)$

$-2y = x - 1$

$x + 2y - 1 = 0$

6) $y = \arcsin 2x$

$2x = \sin y$

$x = \frac{1}{2}\sin y$

$\frac{dx}{dy} = \frac{1}{2}\cos y$

$\frac{dy}{dx} = \frac{2}{\cos y}$

$\sin^2 y + \cos^2 y \equiv 1$

$(2x)^2 + \cos^2 y \equiv 1$

$\cos^2 y = 1 - 4x^2$

$\cos y = \sqrt{1-4x^2}$

$\rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$

section 2

(1) a) $\frac{dy}{dx} - 2x + 1 = 0$

$\frac{dy}{dx} = 2x - 1$

$y = \int 2x - 1\,dx$

$y = x^2 - x + c$

b) $\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y = \pm\sqrt{x^2 + 2c}$

c) $\frac{dy}{dx} = 5xy$

$\int \frac{dy}{y} = \int 5x\,dx$

$\ln y = \frac{5}{2}x^2 + c$

$y = e^{\frac{5}{2}x^2 + c} = Ae^{\frac{5}{2}x^2}$

d) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$

$\int \cos y\,dy = \int \sin x\,dx$

$\sin y = -\cos x + c$

$y = \sin^{-1}(-\cos x + c)$

(14)

(2) a) $\frac{dy}{dx} + 9x^2 - 2 = 0$

$y = \int 2 - 9x^2\,dx$

$y = 2x - 3x^3 + c$

Substituting (2, −10):

$-10 = 4 - 24 + c$

$\Rightarrow c = 10$

Particular solution: $y = 2x - 3x^3 + 10$

b) $\frac{dy}{dx} = x(y + 3)$

$\int \frac{dy}{y+3} = \int x\,dx$

$\ln|y + 3| = \frac{1}{2}x^2 + c$

$y = e^{\frac{1}{2}x^2 + c} - 3$

c) $2x\frac{dy}{dx} - y = 0$

$\frac{dy}{dx} = \frac{y}{2x}$

$\int \frac{1}{y}dy = \frac{1}{2}\int -\frac{1}{x}dx$

$\ln y = \frac{1}{2}\ln x + c = \ln(A\sqrt{x})$

$y = A\sqrt{x}$

Substituting(1, 2):

$2 = A\sqrt{1}$

$\Rightarrow A = 2$

Particular solution: $y = 2\sqrt{x}$

(13)

(3) a) $\frac{dS}{dt} \propto t$

$\Rightarrow \frac{dS}{dt} = kt$

b) $\frac{dD}{dP} = \frac{k}{P}$

c) $\frac{dI}{dR} \propto 10^R$

$\Rightarrow \frac{dI}{dR} = k10^R$

(3)

Question Number	Scheme		Marks	
(4)	$\int y\,dy = \int \frac{3}{\cos^2 x}dx$ $= \int 3\sec^2 x\,dx$	Can be implied. Ignore integral signs	B1	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	(5)
				[5]

Question	Scheme	Notes	Marks
5	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) Way 1	$\int \frac{1}{x}\,dx = \int -\frac{5}{2}\,dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give either $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ or $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no incorrect working seen	A1 cso
			[4]
(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of either $x = \pm\lambda e^{\pm\mu t} \pm \beta$ or $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ or $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ or $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449...\text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. $(A \in \square, t > 0)$	dM1
	$\Rightarrow t = 632.8006... = 633$ (to the nearest minute)	awrt 633 or 10 hours and awrt 33 minutes	A1 cso
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

6

a $\frac{dP}{dt} = \frac{25\pi}{7}\cos\left(\frac{\pi t}{14}\right)$

$\Rightarrow P = \frac{14}{\pi}\frac{25\pi}{7}\sin\left(\frac{\pi t}{14}\right) + c$

$P = 50\sin\left(\frac{\pi t}{14}\right) + c$

b $50 = 50\sin\left(\frac{\pi \times 0}{14}\right) + c$

$\Rightarrow c = 50$

$P = 50\sin\left(\frac{\pi t}{14}\right) + 50$

c $P = 50\sin\left(\frac{4\pi}{14}\right) + 50$

$P = 89.0...$

So there is 89% visible on day 4.

d $100 = 50\sin\left(\frac{\pi t}{14}\right) + 50$

$\sin\left(\frac{\pi t}{14}\right) = 1$

$\frac{\pi t}{14} = \frac{\pi}{2}$

$t = 7$

9

7

a $\frac{dw}{dt} \propto (2000 - w)$

$\frac{dw}{dt} = k(2000 - w)$

b $\frac{dw}{dt} = k(2000 - w)$

$\int \frac{dw}{(2000 - w)} = \int k\,dt$

$-\ln(2000 - w) = kt + c$

$t = 0$, $w = 50$ gives

$c = -\ln I(2000 - 50)$

$= -\ln 1950$

$= -7.58$ (3sf)

c $t = 60$, $w = 1990$ gives

$-\ln(2000 - 1990) = 60k - \ln 1950$

$k = \frac{1}{60}\ln\left(\frac{1950}{2000 - 1990}\right)$

$= \frac{1}{60}\ln 195$

$= 0.0879$ (3sf)

d $-\ln(2000 - w) = 0.0879t - 7.58$

$(2000 - w) = e^{-0.0879t + 7.58}$

$w = 2000 - e^{-0.0879t + 7.58}$

e $1500 = 2000 - e^{-0.0879t + 7.58}$

$e^{-0.0879t + 7.58} = 500$

$-0.0879t + 7.58 = \ln 500$

$t = \frac{\ln 500 - 7.58}{-0.0879}$

$= 15.533...$

On the 15th day their weight will exceed 1500 g.