Godalming College

# Parametrics SOLUTIONS

Section 1:

| | | | |
|---|---|---|---|
| 1(a) | $\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \ldots\right\}$ | | |
| | Either $(-4)k = -6$ or $(1+kx)^{-4} = 1 + (-4)(kx)$ | | |
| | leading to $k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$ | | |
| (b) | $\frac{(-4)(-5)}{2}(k)^2$ | Either $\frac{(-4)(-5)}{2!}$ or $(k)^2$ or $(kx)^2$ | |
| | | Either $\frac{(-4)(-5)}{2!}(k)^2$ or $\frac{(-4)(-5)}{2!}(kx)^2$ | |
| | $\left\{A = \frac{(-4)(-5)}{2!}\left(\frac{3}{2}\right)^2\right\} \Rightarrow A = \frac{45}{2}$ | $\frac{45}{2}$ or 22.5 | |

| | | |
|---|---|---|
| 2(a) | $R = \sqrt{5}$ | |
| | $\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 26.57°$ | |
| | | |
| (b) | $\frac{2}{2\cos\theta - \sin\theta - 1} = 15 \Rightarrow \frac{2}{\sqrt{5}\cos(\theta + 26.6°) - 1} = 15$ | |
| | $\Rightarrow \cos(\theta + 26.6°) = \frac{17}{15\sqrt{5}} = (awrt\ 0.507)$ | |
| | $\theta + 26.57° = 59.54°$ | |
| | $\Rightarrow \theta = awrt\ 33.0°$ or $awrt\ 273.9°$ | |
| | $\theta + 26.6° = 360° - \text{their}'59.5°'$ | |
| | $\Rightarrow \theta = awrt\ 273.9°$ and $awrt\ 33.0°$ | |
| | | |
| (c) | $\theta - \text{their } 26.57° = \text{their } 59.54° \Rightarrow \theta = \ldots$ | |
| | $\theta = \text{awrt } 86.1°$ | |
| | | |

| | | |
|---|---|---|
| 3(a) | $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$ | |
| | $= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ | B1 B1 |
| | | (2) |
| (b) | $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{4}{4\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$ | |
| | $= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$ | M1 |
| | Using $1-\cos^2\theta = \sin^2\theta$ $= \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta}$ | M1 |
| | $= \dfrac{1}{\cos^2\theta} = \sec^2\theta$ | M1 A1* |
| | | (4) |
| (c) | $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$ | M1 |
| | $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ | A1 A1 |
| | | (3) |
| | | (9 marks) |

(61)

# Parametric equations

## Section 2: Using parametric equations

### Solutions to Exercise

1. (i) $x = 1 - t \Rightarrow t = 1 - x$ ✓

$y = t^2 - 4$

$= (1-x)^2 - 4$ ✓

$= 1 - 2x + x^2 - 4$

$= x^2 - 2x - 3$ ✓ (3)

(ii) $y = \frac{1}{t} \Rightarrow t = \frac{1}{y}$ ✓

$x = 2t^2 \Rightarrow x = 2\left(\frac{1}{y}\right)^2 = \frac{2}{y^2}$ ✓ (3)

$xy^2 = 2$ ✓

(iii) $x = 2\cos\theta + \sin\theta \Rightarrow x^2 = 4\cos^2\theta + 4\cos\theta\sin\theta + \sin^2\theta$

$y = \cos\theta - 2\sin\theta \Rightarrow y^2 = \cos^2\theta - 4\cos\theta\sin\theta + 4\sin^2\theta$ ✓

Adding: $x^2 + y^2 = 5\cos^2\theta + 5\sin^2\theta = 5(\cos^2\theta + \sin^2\theta)$ ✓ (3)

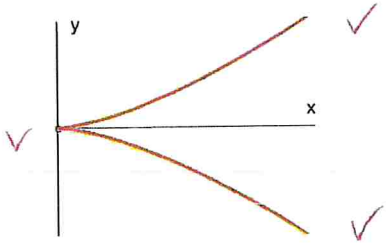
$x^2 + y^2 = 5$ ✓

2. (i)

| $t$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $x$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |
| $y$ | -27 | -8 | -1 | 0 | 1 | 8 | 27 |

✓✓ ✓✓ (4)

(ii)




✓ ✓ ✓ (3)

(iii) $x = t^2 \Rightarrow t = \pm\sqrt{x}$ ✓

$y = t^3 = (\pm\sqrt{x})^3$ ✓

$y = \pm x^{\frac{3}{2}}$ ✓ (3)

3. (i)

| $\theta$ | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\frac{11\pi}{12}$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 3 | 2.9 | 2.6 | 2.1 | 1.5 | 0.8 | 0 | -0.8 | -1.5 | -2.1 | -2.6 | -2.9 | -3 |
| $y$ | 0 | 0.5 | 1 | 1.4 | 1.7 | 1.9 | 2 | 1.9 | 1.7 | 1.4 | 1 | 0.5 | 0 |

✓✓ ✓✓

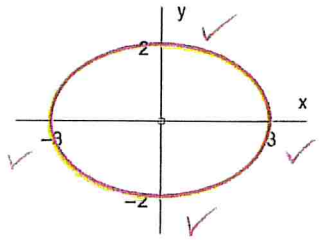
(ii)

| $\theta$ | $\frac{13\pi}{12}$ | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{17\pi}{12}$ | $\frac{3\pi}{2}$ | $\frac{19\pi}{12}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | $\frac{23\pi}{12}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | -2.9 | -2.6 | -2.1 | -1.5 | -0.8 | 0 | 0.8 | 1.5 | 2.1 | 2.6 | 2.9 | 3 |
| $y$ | -0.5 | -1 | -1.4 | -1.7 | -1.9 | -2 | -1.9 | -1.7 | -1.4 | -1 | -0.5 | 0 |

✓✓ ✓✓

(8)

(iii)




✓ ✓ ✓ ✓

(4)

(iv) $x = 3\cos\theta \Rightarrow \frac{x}{3} = \cos\theta \Rightarrow \frac{x^2}{9} = \cos^2\theta$ ✓

$y = 2\sin\theta \Rightarrow \frac{y}{2} = \sin\theta \Rightarrow \frac{y^2}{4} = \sin^2\theta$ ✓

Adding: $\frac{x^2}{9} + \frac{y^2}{4} = \cos^2\theta + \sin^2\theta$ ✓

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ ✓

(4)

4. (i) Where curve meets the x-axis, $t - t^2 = 0$ ✓

$t(t-1) = 0$

$t = 0$ or $t = 1$ ✓ ✓

(3)

5. (i) The curve is undefined for $t = 0$. ✓

(ii) When $x = 0$, $t - \frac{1}{t} = 0 \Rightarrow t = \frac{1}{t} \Rightarrow t^2 = 1 \Rightarrow t = \pm 1$ ✓

When $t = 1$, $y = 2\left(1 + \frac{1}{1}\right) = 4$

When $t = -1$, $y = 2\left(-1 + \frac{1}{-1}\right) = -4$ ✓

so the curve crosses the $y$-axis at $(0, 4)$ and $(0, -4)$. ✓ ✓

When $y = 0$, $t + \frac{1}{t} = 0 \Rightarrow t = -\frac{1}{t} \Rightarrow t^2 = -1$ ✓

so the curve does not cross the $x$-axis. ✓ (7)

(iii) $x = t - \frac{1}{t} \Rightarrow 2x = 2t - \frac{2}{t}$

$y = 2\left(t + \frac{1}{t}\right) \Rightarrow y = 2t + \frac{2}{t}$ ✓

Adding: $2x + y = 4t \Rightarrow t = \frac{2x + y}{4}$

Substituting into $x = t - \frac{1}{t}$: ✓

$x = \frac{2x + y}{4} - \frac{4}{2x + y}$ ✓

$4x(2x + y) = (2x + y)^2 - 16$

$8x^2 + 4xy = 4x^2 + 4xy + y^2 - 16$

$y^2 = 4x^2 + 16$ ✓ (4)

6. (i)

$x = 4t^2 \Rightarrow t = \frac{1}{2}\sqrt{x}$ ✓

$y = 2t(1 - t^2)$

$= 2 \times \frac{1}{2}\sqrt{x}\left(1 - \frac{1}{4}x\right)$ ✓

$= \sqrt{x}\left(1 - \frac{1}{4}x\right)$ ✓ (3)

7. (i) $x = 16t\cos\theta \Rightarrow t = \dfrac{x}{16\cos\theta}$ ✓

$y = 16\sin\theta \times \dfrac{x}{16\cos\theta} - 5\left(\dfrac{x}{16\cos\theta}\right)^2$ ✓

$y = x\tan\theta - \dfrac{5x^2}{256}\sec^2\theta$ ✓

$y = x\tan\theta - \dfrac{5x^2}{256}(1 + \tan^2\theta)$ (5)

✓ ✓

(ii) The ball bounces when $y = 0$. ✓

$16t\sin\theta - 5t^2 = 0$

$16t\sin 30^\circ - 5t^2 = 0$ ✓

$8t - 5t^2 = 0$

$t(8 - 5t) = 0$

$t = 0$ or $t = 1.6$ ✓

The ball bounces when $t = 1.6$ (4)

$x = 16 \times 1.6\cos 30^\circ = 22.2$ ✓

61