

## Mechanics 10 – Dynamics, inclined planes, connected particles.

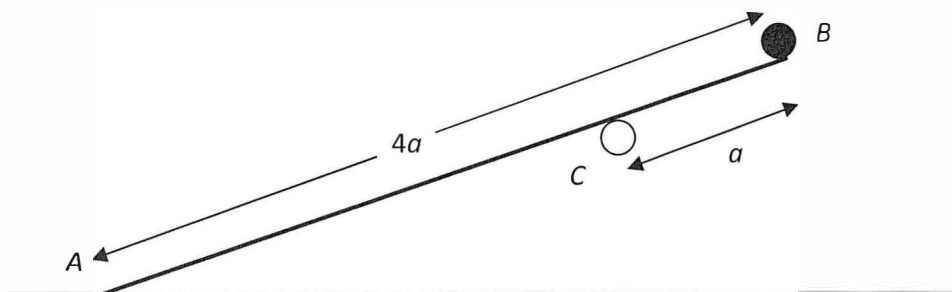
Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics.

Please **complete** all questions.

1)

Figure 1



A wooden plank  $AB$  has mass  $4m$  and length  $4a$ . The end  $A$  of the plank lies on rough horizontal ground. A small stone of mass  $m$  is attached to the plank at  $B$ . The plank is resting on a small smooth horizontal peg  $C$ , where  $BC = a$ , as shown in Figure 2. The plank is in equilibrium making an angle  $\alpha$  with the horizontal, where  $\tan \alpha = \frac{3}{4}$ . The coefficient of friction between the plank and the ground is  $\mu$ . The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle.

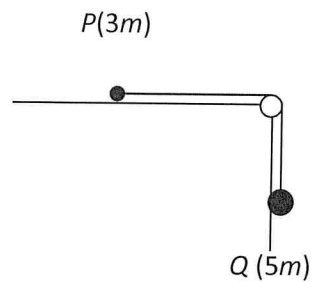
- (a) Show that the reaction of the peg on the plank has magnitude  $\frac{16}{5} mg$ , (3)
- (b) Show that  $\mu \geq \frac{48}{61}$ . (6)
- (c) State how you have used the information that the peg is smooth. (1)
- 2) The daily maximum temperature  $T$  at a weather station in August is found over several years to have a mean of  $22.1^\circ\text{C}$  and a standard deviation of  $3.86^\circ\text{C}$ .  $T$  is modelled by the Normal distribution.
- Last year, the maximum daily temperature in a particular week had a mean of  $25.6^\circ\text{C}$ .
- a) Test whether this week was significantly warmer than usual using a 1% significance level.
- b) Find the critical region for the test statistic when using a 10% significance level.

Section 2 – Consolidation of this week's topic.  
Please complete all questions.

- 1) A block of mass 7kg lies on a rough slope inclined at  $35^\circ$  to the horizontal. A string is attached to the block and is pulled with a force of 50N up the slope. The coefficient of friction between the mass and the slope is 0.1. Find the acceleration of the block (to 2 sig figs). (4)
- 2) A box of mass 5kg lies on a rough slope inclined at  $40^\circ$  to the horizontal. A light inextensible string is attached to the box. The string passes over a smooth pulley fixed to the top of the slope. The other end of the string is attached to a box of mass 6kg which hangs vertically, 1m above the floor.
- a) The 6kg mass is released from rest and after 2 sec, it hits the floor. Find the coefficient of friction between the 5kg mass and the slope. (6)
- b) Explain how you have used the fact that
- i) the string is light
  - ii) the string is inextensible
  - iii) the pulley is smooth (3)
- 3) A boy kicks a block of mass 2kg so that it slides up a rough plane inclined at  $30^\circ$  to the horizontal. The block has an initial speed of  $3\text{ms}^{-1}$ . The coefficient of friction between the block and the plane is  $\frac{1}{2}$ .
- a) Show that after it has been kicked, the deceleration of the block up the slope is  $9.1\text{ms}^{-2}$  to 2 sig figs.
- b) Find the distance travelled by the block before it comes to instantaneous rest.
- c) Find the acceleration of the block as it slides back down the slope.
- d) Show that the block will have a speed of  $0.8\text{ms}^{-1}$  to 1 sig fig as it returns to the point where it was kicked. (10)

4)

**Figure 2**



Two particles  $P$  and  $Q$  have masses  $3m$  and  $5m$  respectively. They are connected by a light inextensible string which passes over a small smooth light pulley fixed at the edge of a rough horizontal table. Particle  $P$  lies on the table and particle  $Q$  hangs freely below the pulley, as shown in Fig. 2. The coefficient of friction between  $P$  and the table is  $0.6$ . The system is released from rest with the string taut. For the period before  $Q$  hits the floor or  $P$  reaches the pulley,

(a) write down an equation of motion for each particle separately (4)

(b) find, in terms of  $g$ , the acceleration of  $Q$  (4)

(c) find, in terms of  $m$  and  $g$ , the tension in the string. (2)

When  $Q$  has moved a distance  $h$ , it hits the floor and the string becomes slack. Given that  $P$  remains on the table during the subsequent motion and does not reach the pulley,

(d) find, in terms of  $h$ , the distance moved by  $P$  after the string becomes slack until  $P$  comes to rest. (6)

5)

Figure 3

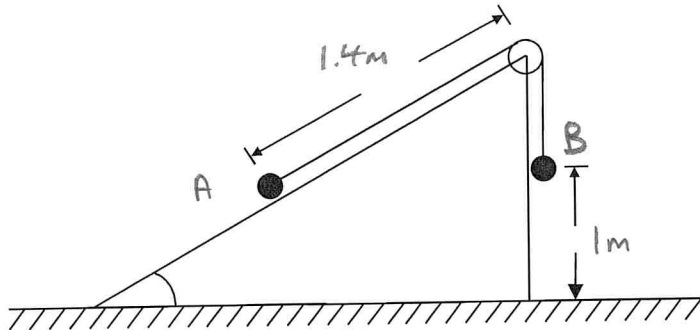


Figure 3 shows two particles  $A$  and  $B$ , of mass  $m$  kg and  $0.4$  kg respectively, connected by a light inextensible string. Initially  $A$  is held at rest on a fixed smooth plane inclined at  $30^\circ$  to the horizontal. The string passes over a small light smooth pulley  $P$  fixed at the top of the plane. The section of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. The particle  $B$  hangs freely below  $P$ . The system is released from rest with the string taut and  $B$  descends with acceleration  $\frac{1}{5}g$ .

- (a) Write down an equation of motion for  $B$ . (2)
- (b) Find the tension in the string. (2)
- (c) Prove that  $m = \frac{16}{35}$ . (4)
- (d) State where in the calculations you have used the information that  $P$  is a light smooth pulley. (1)

On release,  $B$  is at a height of one metre above the ground and  $AP = 1.4$  m. The particle  $B$  strikes the ground and does not rebound.

- (e) Calculate the speed of  $B$  as it reaches the ground. (2)
- (f) Show that  $A$  comes to rest as it reaches  $P$ . (5)

**Total Mark: 55**