Mechanics 15 – Vector Calculus

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1)

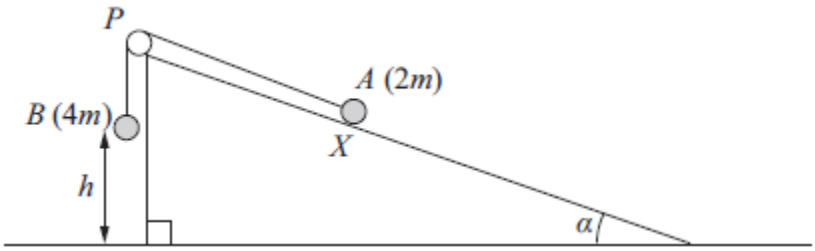

Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same, **(1)**

(b) write down an equation of motion for each particle, **(4)**

(c) find the acceleration of each particle. **(5)**

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(d) find the distance XY in terms of h. **(6)**

2) A particle P moves on the x-axis. The acceleration of P at time t seconds is $(t - 4)$ m s^{-2} in the positive x-direction. The velocity of P at time t seconds is v m s^{-1}. When $t = 0$, $v = 6$.

Find

(a) v in terms of t, **(4)**

(b) the values of t when P is instantaneously at rest, **(3)**

(c) the distance between the two points at which P is instantaneously at rest. **(4)**

Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) A particle moves on a plane such that its position at time t is given by

$$\mathbf{r} = (3t - 2)\mathbf{i} + (4t - 2t^2)\mathbf{j} \text{ m}$$

a) Write expressions for the velocity and acceleration of the particle at time t. **(4)**
b) Work out the initial speed of the particle. **(2)**
c) At what time is the particle moving parallel to the x-axis? **(2)**
d) Is the particle ever stationary? Give a reason for your answer. **(2)**

2) At time t, a particle has position given by $\mathbf{r} = (2t - 1 + \cos t)\mathbf{i} + (\sin 2t)\mathbf{j}$. The particle starts at the origin.

a) Work out the value of t for which it next touches the x-axis. **(2)**

b) For that value of t, work out its instantaneous velocity and acceleration, showing your working. **(4)**

3) The force acting on a particle of mass 500kg at time t sec is given by $\mathbf{F} = (2000t\mathbf{i} - 4000\mathbf{j})$ N. Initially the particle is at the origin and travelling with velocity $10\mathbf{i}$ ms^{-1}.

Work out a) the speed of the particle when t = 2. **(5)**

b) the distance of the particle from the origin at this time. **(4)**

4) A particle starts moving from the point with position vector (2**i** + 3**j**) m and has a velocity given by **v** = (10e^{-t}**i** + 2**j**) ms^{-1}

Find a) the acceleration when t = 1 **(3)**

b) the position vector when t = 1 **(4)**

5) An object moves on a plane so that its acceleration at time t sec is given by

a = (-4cos2t**i** – 4sin2t**j**) ms^{-2}.

It is initially at the point (1,0) and travelling at 2ms^{-1} in the positive y-direction.

a) Show that the object moves with constant speed. **(3)**

b) Work out the distance of the object from the origin at time t and hence describe the path of the object. **(4)**

6) Two boats P and Q, move on the ocean, assumed to be a plane, with **i** and **j** as unit vectors acting East and North respectively. Initially P has velocity (2**i** – 5**j**) ms^{-1} and Q is travelling North at 2ms^{-1}. After t sec, each boat has an acceleration of magnitude t ms^{-2}. For P, this acceleration is towards the North East and for Q, it is towards the South East. Show that the acceleration of P is ((√2/2)t**i** + (√2/2)t**j**)ms^{-2} and find a similar expression for the acceleration of Q. Hence find the value of t for which the boats have the same speed.

(6)

7) A particle of mass 2kg is acted upon by the two forces $F_1 = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix}, F_2 = \begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix}$.

Given that the magnitude of the acceleration is $\sqrt{26}\, ms^{-2}$ find the possible vaues of x **(4 marks)**

8) A particle is in equilibrium when it is acted upon by the three forces

$F_1 = 3i + 2j - k,\ F_2 = i - 4j + 5k$ and $F_3 = 3i + j - 2k$

a) Find the exact magnitude of the resultant force **(2 marks)**

b) Given that the paricle accelerates with magnitude $\sqrt{6}ms^{-2}$ find the mass. **(1 mark)**

c) The third force is removed. Find:

i) the magnitude of the acceleration **(3 marks)**

ii) the direction of the acceleration, giving your answers as angles against the axes **(4 marks)**

Total mark: 59

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