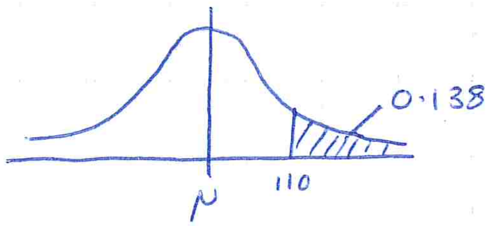
Statistics 3 - Normal distribution (Hyp testing)

Section 1

1, $B \sim N(\mu, 7.2^2)$ $P(B > 110) = 0.138$

$p = 1 - 0.138 = 0.862$

$z = 1.0893$

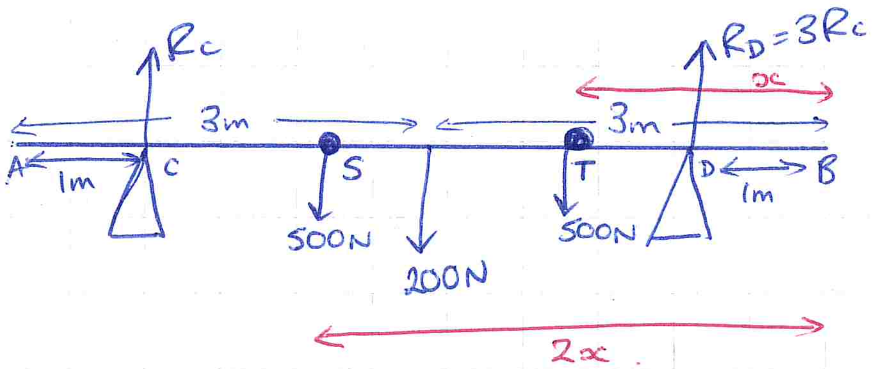


$z = \frac{x - \mu}{\sigma}$

$1.0893 = \frac{110 - \mu}{7.2}$

$\underline{\mu = 102.2}$

2,



Resolving forces $R_c + R_D = 500 + 200 + 500$

$R_c + 3R_c = 1200$

$4R_c = 1200$

$R_c = 300N \quad \therefore R_D = 900N.$

Moments about B

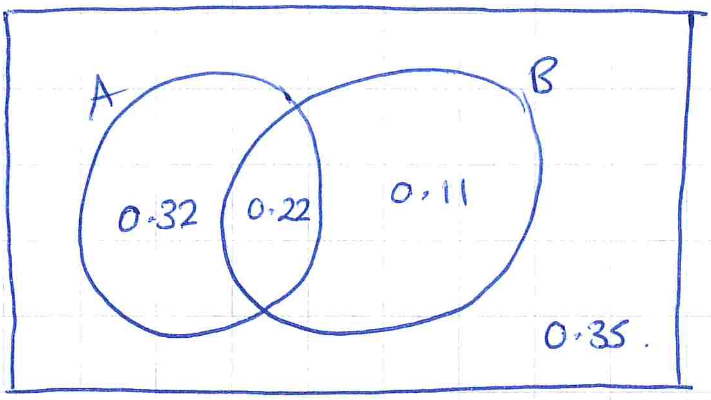
$1 \times R_D + 5 \times R_c = 500 \times x + 200 \times 3 + 500 \times 2x$

$900 + 1500 = 500x + 600 + 1000x$

$1800N = 1500x$ $x = 1.2m$

$\underline{x = 1.2m}$

3, a,



$0.65 - 0.32 - 0.11$

$= 0.22$

b, $P(A) = 0.54$, $P(B) = 0.33$

c, $P(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{0.32}{0.67} = 0.4776 = \dfrac{32}{67}$

d, If A & B are independent $P(A/B') = P(A)$

$0.4776 \neq 0.54$ ∴ not indep.

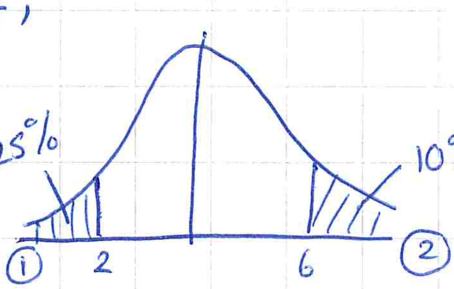
(or $P(A \cap B) = P(A) \times P(B)$

$0.22 \neq 0.54 \times 0.33$

$0.22 \neq 0.1782$

∴ not indep)

4,



① $x = 2$, $p = 0.25$, $z = -0.6745$

$-0.6745 = \dfrac{2 - \mu}{\sigma}$

$-0.6745\sigma = 2 - \mu$.

② $x = 6$, $p = 0.9$ $(1-0.1)$, $z = 1.2816$

$$1.2816 = \frac{6-\mu}{\sigma}$$

② $1.2816\sigma = 6 - \mu$

① $-0.6745\sigma = 2 - \mu$ $-$

$1.9561\sigma = 4$

$\sigma = 2.045$

Sub into ② $1.2816 \times 2.045 = 6 - \mu$, $\mu = 3.38$

b, $X \sim N(3.38, 2.045^2)$

$P(X < 1) = 0.1222 = 12.2\%$.

c, The percentage predicted by the model is a lot larger than the observed ~~p~~percentage, so this suggests the Normal distribution is not a great Model in this situation

Section 2

1, $H_0: \mu = 80$, $H_1: \mu > 80$

$\bar{X} \sim N(80, \frac{15^2}{100})$ $(\sigma = 1.5)$

$P(\bar{X} > 83) = 0.02275$
$= 2.275\%$

$2.275\% < 5\%$ ∴ Reject H_0,
There is evidence to support the mangers claim.

Alternatively

Critical value $= 1.645$

$z = 2$

$2 > 1.645$ 5%.

∴ Reject H_0,
There is evidence to support the managers claim.

2, $W \sim N(\mu, 20\cdot25)$ $H_0: \mu = 50$ $H_1: \mu \neq 50$ 5% level 2 tailed test $\Downarrow$ 2.5%.

$\bar{W} \sim N\left(50, \dfrac{20\cdot25}{81}\right)$

$\sigma = 0\cdot5$ ✓

$p = 2\cdot5\%$ $z = -1\cdot95$ ✓ ...... $p = 97\cdot5\%$ $z = 1\cdot95$ ✓

$-1\cdot95 = \dfrac{W - 50}{0\cdot5}$ $1\cdot95 = \dfrac{W-50}{0\cdot5}$

$W = 49\cdot025$ $W = 50\cdot975$

Critical region $\Rightarrow$ $\bar{W} \leq 49\cdot025$ (or $<$) ✓ $\bar{W} \geq 50\cdot975$ (or $>$) ✓

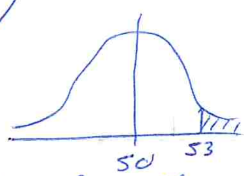
3, $U \sim N(\mu, \sigma^2)$ $H_0: \mu = 50$ $H_1: \mu > 50$ 1%, 1 tailed test. $n = 10$

$\bar{U} \sim N\left(50, \dfrac{\sigma^2}{10}\right)$

$\sigma = \dfrac{\sigma}{\sqrt{10}}$ ✓

Sample mean $= 53$ $\longrightarrow$ H_0 is not rejected

So $P(U > 53) > 1\%$



Critical value for 1% is $z = 2\cdot326$ ✓

So if the z value of 53 is less than 2.326 then H_0 is not rejected.

$2\cdot326 > \dfrac{53 - 50}{\sigma/\sqrt{10}}$ ✓

$\sigma > 4\cdot08$ ✓

4, $\quad n = 4 \qquad \bar{X} \sim \left(500, \dfrac{100^2}{4}\right) \qquad \bar{x} = 435$

$\sigma = 50$

10% level $\qquad H_0 : \mu = 500$

1 tailed test $\qquad H_1 : \mu < 500$

$P(X < 435) = 0.0968 = 9.68\% < 10\%$

$\therefore$ Reject H_0; there is evidence that the no. of visitors has decreased.

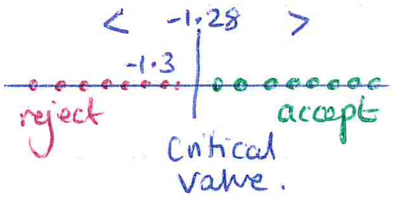
<u>Alternative method</u> (using critical values)

$$z = \frac{435 - 500}{50} = -1.3$$

10% level $\Rightarrow$ Critical value $= -1.28$

$-1.3 < -1.28$

So Reject H_0.



Total = 23 marks