1 Force and momentum

1.1 Momentum and impulse

Learning objectives:

- How do we calculate momentum?
- What is the connection between Newton's first and second laws of motion and momentum?
- What is an impulse, and how is it calculated from a force v. time graph?

Specification reference: 3.4.1

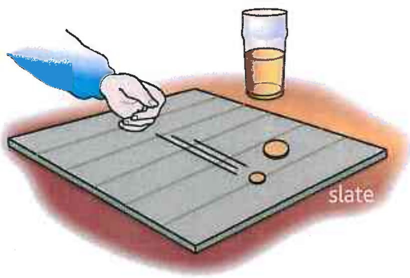

Figure 1 *Momentum games*

Momentum

If you have ever run into someone on the sports field, you will know something about momentum. If the person you ran into was more massive than you, then you probably came off worse than the other person. When two bodies collide, the effect they have on each other depends not only on their initial velocities but also on the mass of each object. You can easily test the idea using coins, as shown in Figure 1. You might already have developed your skill in this area! It is not too difficult to show that when a large coin and a small coin collide, the motion of the small coin is affected more.

Sir Isaac Newton was the first person to realise that a **force** was needed to change the velocity of an object. He realised that the effect of a force on an object depended on its mass as well as on the amount of force. He defined the **momentum** of a moving object as its mass × its velocity and showed how the momentum of an object changes when a force acts on it. In the AS course, you learned that the force needed to give an object a certain acceleration can be calculated from the equation 'force = mass × acceleration'. In the A2 course, we consider the ideas that Newton established in full.

How science works

Universal laws

Although Newton put forward his ideas over 300 years ago, his laws continue to provide the essential mathematical rules for predicting the motion of objects in any situation except inside the atom (where the rules of quantum physics apply) and at speeds approaching the speed of light or in very strong gravitational fields (where Einstein's theories of relativity apply). For example, the launch of a rocket is carefully planned using **Newton's laws of motion** and his **law of gravitation** which we will study in Chapter 4. However, the laws do not for example predict the existence of black holes, a confirmed prediction of Einstein's theory of general relativity. In fact, Einstein showed that his theories of relativity simplify into Newton's laws, where gravity is weak and the speed of objects is much less than the speed of light.

The momentum of an object is defined as its mass × its velocity.

- The unit of momentum is $\mathrm{kg\,m\,s^{-1}}$. The symbol for momentum is p.
- Momentum is a vector quantity. Its direction is the same as the direction of the object's velocity.
- For an object of mass m moving at velocity v, its momentum $p = mv$.

For example, a ball of mass 2.0 kg moving at a velocity of $10\,\mathrm{m\,s^{-1}}$ has the same amount of momentum as a person of mass 50 kg moving at a velocity of $0.4\,\mathrm{m\,s^{-1}}$.

Momentum and Newton's laws of motion

Newton's first law of motion: An object remains at rest or in uniform motion unless acted on by a force.

In effect, Newton's first law tells us that a force is needed to change the momentum of an object. If the momentum of an object is constant, there is no resultant force acting on it. Clearly, if the mass of an object is constant and the object has constant momentum, it follows that the velocity of the object is also constant. If a moving object with constant momentum gains or loses mass, however, its velocity would change to keep its momentum constant. For example, a cyclist in a race who collects a water bottle as he or she speeds past a 'service' point gains mass (i.e. the water bottle) and therefore loses velocity.

Newton's second law of motion: The rate of change of momentum of an object is proportional to the resultant force on it. In other words, the resultant force is proportional to the change of momentum per second.

At AS level, Newton's second law was presented in the form 'force = mass × acceleration'. At A2, we look at how this equation is derived from Newton's second law in its general form as stated above.

Consider an object of constant mass m acted on by a constant force F. Its acceleration causes a change of its speed from initial speed u to speed v in time t without change of direction:

- its initial momentum = mu, and its final momentum = mv
- its change of momentum = its final momentum (mv) – its initial momentum (mu).

According to Newton's second Law, the force is proportional to the change of momentum per second.

Therefore, force $F \propto \dfrac{\text{change of momentum}}{\text{time taken}} = \dfrac{mv - mu}{t}$

$$= \frac{m(v-u)}{t} = ma$$

where $a = \dfrac{v-u}{t}$ = the acceleration of the object.

This proportionality relationship (i.e. $F \propto ma$) can be written as $F = kma$, where k is a constant of proportionality.

The value of k is made equal to 1 by defining the unit of force, **the newton**, as the amount of force that gives an object of mass 1 kg an acceleration of $1\,\mathrm{m\,s^{-2}}$ (i.e. force $F = 1\,\mathrm{N}$, mass $m = 1\,\mathrm{kg}$, acceleration, $a = 1\,\mathrm{m\,s^{-2}}$ so $k = 1$).

Therefore, with $k = 1$, the equation $\boldsymbol{F = ma}$ follows from Newton's 2nd law provided the mass of the object is constant.

In general, the change of momentum of an object may be written as $\Delta(mv)$, where the symbol Δ means 'change of'. Therefore, if the momentum of an object changes by $\Delta(mv)$ in time Δt, the force F on the object is given by the equation

$$F = \frac{\Delta(mv)}{\Delta t}$$

1 **If m is constant,** then $\Delta(mv) = m\Delta v$, where Δv is the change of velocity of the object.

$$\therefore F = \frac{m\Delta v}{\Delta t} = ma \text{ where acceleration } a = \frac{\Delta v}{\Delta t}$$

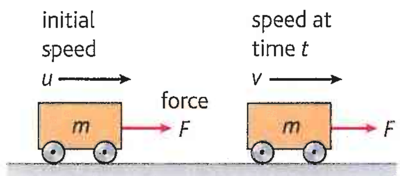

Figure 2 *Force and momentum*

AQA Examiner's tip

The unit of momentum may be either $\mathrm{kg\,m\,s^{-1}}$ or (more neatly) N s.

The equation $F = \dfrac{\Delta(mv)}{\Delta t}$ always applies but $F = ma$ applies only to objects of constant mass.

2 **If m changes at a constant rate** as a result of mass being transferred at constant velocity, then $\Delta(mv) = v\Delta m$, where Δm is the change of mass of the object.

$$\therefore F = \frac{v\Delta m}{\Delta t} \text{ where } \frac{\Delta m}{\Delta t} = \text{change of mass per second.}$$

This form of Newton's second law is used in any situation where an object gains or loses mass continuously.

For example, if a rocket ejects burnt fuel as hot gas from its engine at speed v, the force F exerted by the engine to eject the hot gas is given by

$$F = \frac{v\Delta m}{\Delta t} \text{ where } \frac{\Delta m}{\Delta t} = \text{mass of hot gas lost per second.}$$

An equal and opposite reaction force acts on the jet engine due to the hot gas, propelling the rocket forwards.

The **impulse** of a force is defined as the force × the time for which the force acts. Therefore, for a force F which acts for time Δt,

$$\text{the impulse} = F\Delta t = \Delta(mv)$$

Hence the impulse of a force acting on an object is equal to the change of momentum of the object.

Link

Use of $F = \frac{mv - mu}{t}$ is another way to calculate an impact force.

See *AS Physics* Topic 9.5.

Force–time graphs

Suppose an object of constant mass m is acted on by a constant force F which changes its velocity from initial velocity u to velocity v in time t. As explained earlier in this topic, Newton's second law gives

$$F = \frac{mv - mu}{t}$$

Rearranging this equation gives $Ft = mv - mu$

Figure 3 is a graph of force v. time for this situation. Because force F is constant for time t, the area under the line represents the impulse of the force Ft which is equal to $mv - mu$. In other words,

the area under the line of a force–time graph represents the change of momentum or the impulse of the force.

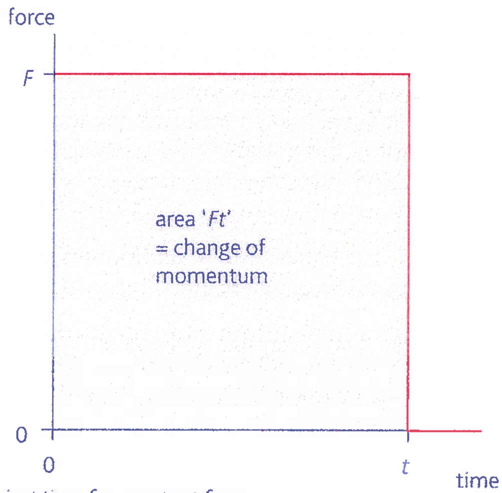

Figure 3 *Force against time for constant force*

Note: The unit of momentum can be given as the newton second (N s) or the kilogram metre per second (kg m s^{-1}). The unit of impulse is usually given as the newton second.

Worked example:

A force of 10 N acts for 20 s on an object of mass 50 kg which is initially at rest.

Calculate:

a the change of momentum of the object,

b the velocity of the object at 20 s.

Solution

a Change of momentum = impulse of the force = Ft = 10 × 20 = 200 N s

b Momentum at 20 s = 200 N s as the object was initially at rest.

$$\therefore \text{Velocity} = \frac{\text{momentum}}{\text{mass}} = \frac{200}{50} = 4.0\,\text{m}\,\text{s}^{-1}$$

Summary questions

1 **a** Calculate the momentum of:

- **i** an atom of mass 4.0×10^{-25} kg moving at a velocity of $3.0 \times 10^{6}\,\text{m}\,\text{s}^{-1}$,
- **ii** a pellet of mass 4.2×10^{-4} kg moving at a velocity of $120\,\text{m}\,\text{s}^{-1}$,
- **iii** a bird of mass 0.56 kg moving at a velocity of $25\,\text{m}\,\text{s}^{-1}$.

b Calculate:

- **i** the mass of an object moving at a velocity of $16\,\text{m}\,\text{s}^{-1}$ with momentum of $96\,\text{kg}\,\text{m}\,\text{s}^{-1}$,
- **ii** the velocity of an object of mass 6.4 kg that has momentum of $128\,\text{kg}\,\text{m}\,\text{s}^{-1}$.

2 A train of mass 24 000 kg moving at a velocity of $15.0\,\text{m}\,\text{s}^{-1}$ is brought to rest by a braking force of 6000 N. Calculate

a the initial momentum of the train,

b the time taken for the brakes to stop the train.

3 An aircraft of total mass 45 000 kg accelerates on a runway from rest to a velocity of $120\,\text{m}\,\text{s}^{-1}$ when it takes off. During this time, its engines provide a constant driving force of 120 kN. Calculate:

a the gain of momentum of the aircraft,

b the 'take off' time.

4 The velocity of a vehicle of mass 600 kg was reduced from $15\,\text{m}\,\text{s}^{-1}$ by a constant force of 400 N which acted for 20 s then by a constant force of 20 N for a further 20 s.

a Sketch the force v. time graph for this situation.

b **i** Calculate the initial momentum of the vehicle.

- **ii** Use the force v. time graph to determine the total change of momentum.
- **iii** Hence show that the final velocity of the vehicle is $1\,\text{m}\,\text{s}^{-1}$.