11.2 Springs

Learning objectives:

- Is there any limit to the linear graph of force against extension for a spring?
- What is the meaning of spring constant, and in what unit is it measured?
- If the extension of a spring is doubled, how much more energy does it store?

Specification reference: 3.2.2

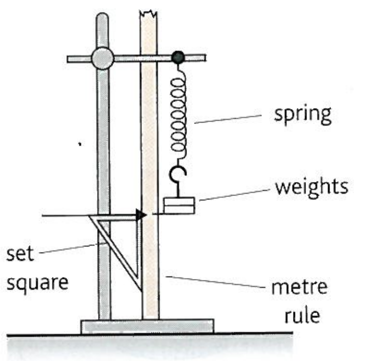

Figure 1 *Testing the extension of a spring*

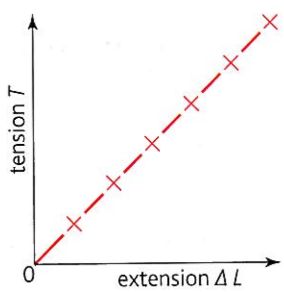

Figure 2 *Hooke's law*

Hooke's law

A stretched spring exerts a pull on the object holding each end of the spring. This pull, referred to as the **tension** in the spring, is equal and opposite to the force needed to stretch the spring. The more a spring is stretched, the greater the tension in it. Figure 1 shows a stretched spring supporting a weight at rest. This arrangement may be used to investigate how the tension in a spring depends on its extension from its unstretched length. The measurements may be plotted on a graph of tension against extension, as shown in Figure 2. The graph shows that the force needed to stretch a spring is proportional to the extension of the spring. This is known as **Hooke's law**, after its discoverer, Robert Hooke, a seventeenth century scientist.

Hooke's law states that the force needed to stretch a spring is directly proportional to the extension of the spring from its natural length.

Hooke's law may be written as

$$\text{Force } F = k\Delta L$$

where k is the spring constant (sometimes referred to as the stiffness constant) and ΔL is the extension from its natural length L.

- The greater the value of k, the stiffer the spring is. The unit of k is $N\,m^{-1}$.
- The graph of F against ΔL is a straight line of gradient k through the origin.
- If a spring is stretched beyond its **elastic limit**, it does not regain its initial length when the force applied to it is removed.
- AS/A level maths students may meet Hooke's law in the form $F = \lambda \Delta L/L$, where L is the unstretched length of the spring and λ $(= kL)$ is the spring modulus. Note that λ is not in the specification for AS/A level physics.

Worked example:

A vertical steel spring fixed at its upper end has an unstretched length of 300 mm. Its length is increased to 385 mm when a 5.0 N weight attached to the lower end is at rest. Calculate:

a the spring constant,

b the length of the spring when it supports an 8.0 N weight at rest.

Solution

a Use $F = k\Delta L$ with $F = 5.0\,N$ and $\Delta L = 385 - 300\,mm = 85\,mm = 0.085\,m$.

Therefore $k = \dfrac{F}{\Delta L} = \dfrac{5.0\,N}{0.085\,m} = 59\,N\,m^{-1}$.

b Use $F = k\Delta L$ with $F = 8.0\,N$ and $k = 59\,N\,m^{-1}$ to calculate ΔL.

$\Delta L = \dfrac{F}{k} = \dfrac{8.0\,N}{59\,N\,m^{-1}} = 0.136\,m$

Therefore the length of the spring $= 0.300\,m + 0.136\,m = 0.436\,m$

Spring combinations

Springs in parallel

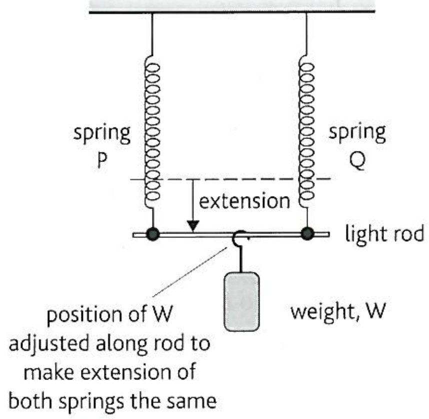

Figure 3 *Two springs in parallel*

Figure 3 shows a weight W supported by means of two springs P and Q in parallel with each other. The extension, ΔL, of each spring is the same. Therefore

- the force needed to stretch P, $F_P = k_P \Delta L$
- the force needed to stretch Q, $F_Q = k_Q \Delta L$

where k_P and k_Q are the spring constants of P and Q, respectively.

Since the weight W is supported by both springs, $W = F_P + F_Q = k_P \Delta L + k_Q \Delta L = k \Delta L$

where the effective spring constant, $k = k_P + k_Q$

Springs in series

Figure 4 shows a weight W supported by means of two springs joined end-on in 'series' with each other. The tension in each spring is the same and is equal to the weight W.

Therefore

- the extension of spring P, $\Delta L_P = \frac{W}{k_P}$
- the extension of spring Q, $\Delta L_Q = \frac{W}{k_Q}$

where k_P and k_Q are the spring constants of P and Q, respectively.

Therefore the total extension, $\Delta L = \Delta L_P + \Delta L_Q = \frac{W}{k_P} + \frac{W}{k_Q} = \frac{W}{k}$

where k, the effective spring constant, is given by the equation

$$\frac{1}{k} = \frac{1}{k_P} + \frac{1}{k_Q}$$

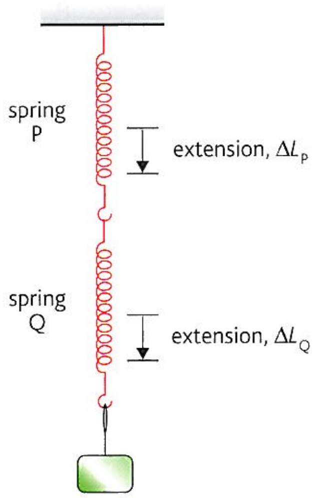

Figure 4 *Two springs in series*

The energy stored in a stretched spring

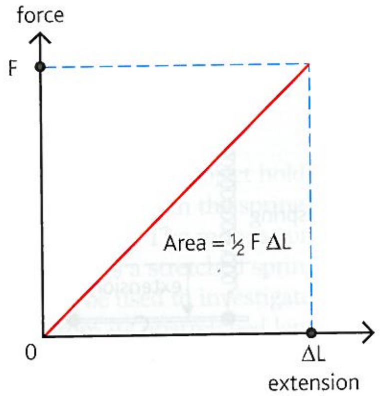

Figure 4 *Energy stored in a stretched spring*

Elastic potential energy is stored in a stretched spring. If the spring is suddenly released, the elastic energy stored in it is suddenly converted to kinetic energy of the spring. As explained in 10.1, the work done to stretch a spring by extension ΔL from its unstretched length $= \frac{1}{2}F\Delta L$, where F is the force needed to stretch the spring to extension ΔL. The work done on the spring is stored as elastic potential energy. Therefore, the elastic potential energy E_p in the spring $= \frac{1}{2}F\Delta L$. Also, since $F = k\Delta L$ where k is the spring constant, then $E_p = \frac{1}{2}k\Delta L^2$.

Elastic potential energy stored in a stretched spring, $E_p = \frac{1}{2}F\Delta L = \frac{1}{2}k\Delta L^2$

AQA Examiner's tip

Energy stored by a spring is $\frac{1}{2}F\Delta L$, not $F\Delta L$

Summary questions

$g = 9.8\,\text{m}\,\text{s}^{-2}$

1. A steel spring has a spring constant of 25 N m^{-1}. Calculate:
   - **a** the extension of the spring when the tension in it is equal to 10 N,
   - **b** the tension in the spring when it is extended by 0.50 m from its unstretched length.
2. Two identical steel springs of length 250 mm are suspended vertically side-by-side from a fixed point. A 40 N weight is attached to the ends of the two springs. The length of each spring is then 350 mm. Calculate:
   - **a** the tension in each spring,
   - **b** the extension of each spring,
   - **c** the spring constant of each spring.
3. Repeat **2a** and **b** for the two springs in 'series' and vertical.
4. An object of mass 0.150 kg is attached to the lower end of a vertical spring of unstretched length 300 mm, which is fixed at its upper end. With the object at rest, the length of the spring becomes 420 mm as a result. Calculate:
   - **a** the spring constant,
   - **b** the energy stored in the spring,
   - **c** the weight that needs to be added to extend the spring to 600 mm.

11.3 Deformation of solids

Learning objectives:

- How is stress related to force, and strain to extension?
- What is meant by tensile?
- Why do we bother with stress and strain, when force and extension are more easily measured?

Specification reference: 3.2.2

Force and solid materials

Look around at different materials and think about the effect of force on each material. To stretch or twist or compress the material, a pair of forces is needed. For example, stretching a rubber band requires the rubber band to be pulled by a force at either end. Some materials, such as rubber, bend or stretch easily. The **elasticity** of a solid material is its ability to regain its shape after it has been deformed or distorted, and the forces that deformed it have been released. Deformation that stretches an object is **tensile**, whereas deformation that compresses an object is **compressive**.

The arrangement shown in Topic 11.2 Figure 1 shows how to test different materials to see how easily they stretch. In each case, the material is held at its upper end and loaded by hanging weights at its lower end. A set square or pointer attached to the bottom of the weights may be used to measure the extension of the material, as the weight of the load is increased in steps then decreased to zero. The extension of the strip of material at each step is its increase of length from its unloaded length. The tension in the material is equal to the weight. The measurements may be plotted as a tension–extension graph, as shown in Figure 1.

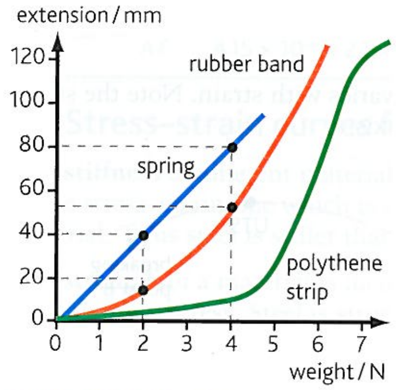

Figure 1 *Typical curves*

- A steel spring gives a straight line, in accordance with Hooke's law (see Topic 11.2).
- A rubber band at first extends easily when it is stretched. However, it becomes fully stretched and very difficult to stretch further when it has been lengthened considerably.
- A polythene strip 'gives' and stretches easily after its initial stiffness is overcome. However, after 'giving' easily, it extends little and becomes difficult to stretch.

Stress and strain

The extension of a wire under tension may be measured using Searle's apparatus, as shown in Figure 2 on the next page (or similar apparatus with a vernier scale). A micrometer attached to the control wire is adjusted so the spirit level between the control and test wire is horizontal. When the test wire is loaded, it extends slightly, causing the spirit level to drop on one side. The micrometer is then readjusted to make the spirit level horizontal again. The change of the micrometer reading is therefore equal to the extension. The extension may be measured for different values of tension by increasing the test weight in steps.

For a wire of length L and area of cross-section A under tension,

- the **tensile stress** in the wire, $\sigma = T/A$, where T is the tension. The unit of stress is the **pascal** (Pa) equal to $1\,\mathrm{N\,m^{-2}}$.
- the **tensile strain** in the wire, $\varepsilon = \Delta L/L$, where ΔL is the extension (increase in length) of the wire. Strain is a ratio and therefore has no unit.