

ENGINEERING PHYSICS

1-1 Angular acceleration

1. $\omega_1 = 0$, $\omega_2 = 12 \text{ rads}^{-1}$, $t = 60 \text{ s}$

(a) $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{12 - 0}{60} = 0.20 \text{ rads}^{-2}$

(b) (i) $\theta = \frac{1}{2}(\omega_1 + \omega_2)t = \frac{1}{2}(0 + 12) \times 60 = 360 \text{ rad}$

OR

$$\omega^2 = \omega_0^2 + 2\alpha\theta \text{ therefore } \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{12^2 - 0}{2 \times 0.20} = 360 \text{ rad}$$

OR

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \times 0.20 \times 60^2 = 360 \text{ rad}$$

(ii) No of turns = $\frac{\theta}{2\pi} = \frac{360}{2\pi} = 57.295\dots = 57$

2. $f = 12 \text{ Hz}$, $\omega_2 = 0 \text{ rads}^{-1}$, $t = 50 \text{ s}$

(a) $\omega_1 = 2\pi f = 2\pi \times 12 = 75.398 = 75 \text{ rads}^{-1}$ to 2 sf ($24\pi \text{ rads}^{-1}$)

(b) $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0 - 75}{50} = -1.507\dots \text{ rads}^{-2}$ when it is decelerating

Angular deceleration = $+1.5 \text{ rads}^{-2}$ to 2 sf

(c) Angle turned through = $\theta = \frac{1}{2}(\omega_1 + \omega_2)t = \frac{1}{2}(75 + 0) \times 50 (= 1875 \text{ rad})$

$$\text{No of turns} = \frac{\theta}{2\pi} = \frac{\frac{1}{2}(75 + 0) \times 50}{2\pi} = 298.41\dots = 300$$

3. $\omega_1 = 0$,

$\omega_2 = 1100 \text{ revolutions per second} = 2\pi \times 1100 = 2200\pi = 6911.5 = 6910 \text{ rads}^{-1}$ to 3 sf

$t = 50 \text{ s}$

(a) $\theta = \frac{1}{2}(\omega_1 + \omega_2)t = \frac{1}{2}(0 + 6910) \times 50 = 1.73 \times 10^5 \text{ rad}$

(b) No of turns = $\frac{\theta}{2\pi} = \frac{1.73 \times 10^5}{2\pi} = 2.75 \times 10^4$

4. $d = 0.45 \text{ m}$, so $r = 0.225 \text{ m}$, $u = 24 \text{ ms}^{-1}$, $v = 0 \text{ ms}^{-1}$, $s = 60 \text{ m}$

Circumference = $2\pi r = 2\pi \times 0.225 = 1.4137\dots \text{ m}$

(a) $v = r\omega$ therefore $\omega = \frac{v}{r} = \frac{24}{0.225} = 107 \text{ rads}^{-1}$

(b) (i) $s = \frac{1}{2}(u + v)t$ therefore $t = \frac{2s}{u+v} = \frac{2 \times 60}{24+0} = 5.0 \text{ s}$

OR (more long-winded)

$$v^2 = u^2 + 2as \text{ therefore } a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 24^2}{2 \times 60} = -4.8 \text{ ms}^{-2}$$

$$v = u + at \text{ therefore } t = \frac{v - u}{a} = \frac{0 - 24}{-4.8} = 5.0 \text{ s}$$

$$(ii) \quad \text{no of turns} = \frac{s}{2\pi r} = \frac{60}{1.4137\dots} = 42.6$$

OR

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t = \frac{1}{2}(107 + 0) \times 5 = 267.5 \text{ rad}$$

$$\text{no of turns} = \frac{267.5}{2\pi} = 42.6$$

(c) angular speed one second before it stopped = angular speed at 4.0 s

$$\omega_0 = 107 \text{ rad s}^{-1} \text{ from (a)}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0 - 107}{5.0} = -21.4 \text{ rads}^{-2}$$

$$\omega = \omega_0 + \alpha t = 107 + (-21.4 \times 4.0) = 107 - 85.6 = 21.4 \text{ rads}^{-1}$$

OR

$$a = \frac{v - u}{t} = \frac{0 - 24}{5} = -4.8 \text{ ms}^{-2}$$

$$v = u + at = 24 + (-4.8 \times 4) = 4.8 \text{ ms}^{-1}$$

$$\omega = \frac{v}{r} = \frac{4.8}{0.225} = 21.3 \text{ rads}^{-1}$$

5. (a) From fig 2 the maximum angular velocity is when the gradient is steepest, so taking a tangent

$$\omega = \frac{30 - 0}{3 - 0.86} = \frac{30}{2.2} = 14.01869\dots = 14 \text{ rads}^{-1} \text{ this figure will vary significantly depending on how the tangent to a not very accurate graph is drawn.}$$

(b) (i) From fig 3 the maximum angular acceleration is when the gradient is steepest, so taking a tangent to the first section

$$\alpha = \frac{12 - 0}{2.36 - 0} = \frac{12}{2.36} = 5.0847\dots = 5.1 \text{ rads}^{-2}$$

(When decelerating the maximum value is

$$\alpha = \frac{0 - 12}{4.46 - 2.36} = \frac{-12}{2.1} = -5.714\dots = -5.7 \text{ rads}^{-2})$$

(ii) The angular displacement can be estimated from the area under the line

Each 'square' is $2 \times 1 = 2 \text{ rad}$

No of 'squares' is approx. 14.5 giving $13.5 \times 2 = 27 \text{ rad} = 30 \text{ rad to 1 sf}$ (15 'squares' gives 30 rad)

$$\text{no of turns} = \frac{29}{2\pi} = 4.6 \quad (30 \text{ rad gives } 4.8)$$