

ENGINEERING PHYSICS

1-3 Rotational kinetic energy

1. $I = 0.048 \text{ kgm}^2$, $\omega_1 = 20 \text{ rads}^{-1}$

(a) rotational k.e. = $\frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.048 \times 20^2 = 9.6 \text{ J}$

(b) $T = ?$, $\omega_1 = 0 \text{ rads}^{-1}$, $\omega_2 = 20 \text{ rads}^{-1}$, $t = 5.0 \text{ s}$

(i) $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{20 - 0}{5.0} = 4.0 \text{ rads}^{-2}$

$T = \alpha I = 4.0 \times 0.048 = 0.192 \text{ Nm}$

(ii) $\theta = \frac{1}{2} (\omega_1 + \omega_2)t = \frac{1}{2} (0 + 20) \times 5 = 50 \text{ rad}$

OR

$W = T\theta$ therefore $\theta = \frac{W}{T}$ ($W = \text{k.e. from (a)}$)

$$= \frac{9.6}{0.192}$$
$$= 50 \text{ rad}$$

OR

$\omega^2 = \omega_0^2 + 2\alpha\theta$ therefore $\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{20^2 - 0}{2 \times 4.0} = 50 \text{ rad}$

OR

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \times 4.0 \times 5.0^2 = 50 \text{ rad}$

2. $m = 0.65 \text{ kg}$, $s = 1.9 \text{ m}$, $t = 4.6 \text{ s}$, axle diameter, $d = 8.5 \text{ mm}$

(a) p.e. loss = $mg\Delta h = 0.65 \times 9.8 \times 1.0$

$$= 12.103$$
$$= 12 \text{ J to 2 sf}$$

(b) t after release = 14 s , time of fall and rise = 4.6 s to fall and rise

$3 \times 4.6 \text{ s} = 13.8 \text{ s}$ so essentially 14 s after release it is essentially at the bottom of the fall

No of times made by flywheel as mass falls = $\frac{\text{distance fallen by mass}}{\text{circumference of shaft}}$

$$= \frac{1.9}{\pi \times 8.5 \times 10^{-3}}$$
$$= 71.15\dots$$
$$= 71 \text{ to 2 sf}$$

Angle turned through = $2\pi \times 71 = 447 \text{ rad}$

Average angular velocity = $\frac{447}{4.6} = 97 \text{ rads}^{-1}$

Therefore max angular velocity = $2 \times 97 = 194 \text{ rads}^{-1}$

$$\text{Speed of object, } v = r\omega = \frac{8.5 \times 10^{-3}}{2} \times 194 = 0.83 \text{ ms}^{-1}$$

OR

$$\text{Average velocity during fall} = \frac{1.9}{4.6}$$

$$\text{Therefore max velocity at bottom} = \frac{1.9}{4.6} \times 2 = 0.83 \text{ ms}^{-1}$$

$$\text{k.e.} = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.65 \times 0.83^2 = 0.22 \text{ J} = 0.2 \text{ J to 1 sf}$$

$$\text{(c) (i) k.e. of flywheel} = 12.1 - 0.2 = 11.9 \text{ J}$$

(ii) moment of inertia of flywheel:

$$E_K = \frac{1}{2} I\omega^2 \quad \text{therefore } I = \frac{2E_K}{\omega^2} = \frac{2 \times 11.9}{194^2} = 6.3 \times 10^{-4} \text{ kgm}^2$$

3. At the top of the slope the ball possesses p.e.

As it rolls down the slope the p.e. becomes rotational and translational k.e.

As it rolls on the horizontal surface, it loses the rotational and translational k.e. because of frictional forces and the energy becomes internal energy.

$$4. d = 0.31 \text{ m, } t = 0.08 \text{ m, } \rho_{\text{STEEL}} = 7800 \text{ kgm}^{-3}$$

$$\text{(a) (i) } m = \rho V = \rho \frac{\pi d^2 t}{4} = 7800 \times \frac{\pi 0.31^2 \times 0.08}{4} = 47.0975 \dots = 47 \text{ kg to 2 sf}$$

$$\text{(ii) moment of inertia } I = \frac{1}{2} MR^2 = \frac{1}{2} \times 47 \times \left(\frac{0.31}{2}\right)^2 = 0.5657 \dots = 0.57 \text{ kgm}^2$$

(b) (i) 3000 revolutions per minute

$$\omega = \frac{\theta}{t} = \frac{3000 \times 2\pi}{60} = 314.15 \dots = 314 \text{ rads}^{-1}$$

$$\begin{aligned} \text{flywheel k.e.} &= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0.57 \times 314^2 \\ &= 28128 \\ &= 2.8 \times 10^4 \text{ J or 28 kJ to 2 sf} \end{aligned}$$

(ii) time of transfer = 30s

$$\begin{aligned} \text{Average power} &= \frac{\text{energy transfer}}{\text{time}} = \frac{2.8 \times 10^4}{30} \\ &= 940 \text{ W to 2 sf} \end{aligned}$$