ENGINEERING PHYSICS

1-4 Angular Momentum

1. $I = 5.0 \times 10^{-2}$ kgm^2, $r = 0.30$ m, $v = 27$ ms^{-1}

(a) angular momentum = $I\omega$ and $v = r\omega$ so

$$\text{angular momentum} = \frac{Iv}{r} = \frac{5.0 \times 10^{-2}}{0.30}$$

= 4.5 Nms to 2 sf (or kgm^{2}s^{-1})

(b) $u = 27$ ms^{-1}, $v = 0$ ms^{-1}, $\omega_1 = 90$ rads^{-1}, $\omega_2 = 0$ rads^{-1}, $I\omega = 4.5$ Nms, $t = 9.0$ s

$$T = \frac{d(I\omega)}{dt} = I\frac{d\omega}{dt} = I\alpha = 5.0 \times 10^{-2} x \frac{90-0}{9.0}$$

= 0.50 Nm

2. 240 revolutions per minute, $I = 0.044$ kgm^2

(a) $\omega = \frac{\theta}{t} = \frac{240 \times 2\pi}{60} = 25.13... = 25$ rads^{-1} to 2 sf

angular momentum = $I\omega$ = 0.044 x 25.13….

= 1.1058…

= 1.1 Nms to 2 sf

(b) common rate of rotation = 160 revolutions per minute

New $\omega = \frac{\theta}{t} = \frac{160 \times 2\pi}{60} = 16.75 = 17$ rads^{-1} to 2 sf

Angular momentum is conserved so $I_1\omega_1 = (I_1 + I_2)\omega_2$

$1.1 = (0.044 + I_2) \times 16.75$

$1.1 = 0.74 + 16.75I_2$

$$I_2 = \frac{1.1-0.74}{16.75}$$

= 0.02167…

= 0.022 kgm^2

(c) initial k.e. = ½ $I_1\omega_1^2$ = ½ x 0.044 x 25.13… ^{2}

= 13.86 J (using 25 rads^{-1} value of ω gives 13.75 J)

final k.e. = ½ $I_2\omega_2^2$ = ½ x(0.044 + 0.022) x 16.75^2

= 9.258… J

= 9.26 J (using 17 rads^{-1} value of ω gives 9.54 J)

loss in k.e. = 13.86 – 9.26 = 4.6 J (13.75 – 9.54 = 4.21 = 4.2 J)

To get the answer given you need to use the 25 rads^{-1} and 16.75 rads^{-1} values for ω i.e.

loss in k.e. = 13.75 – 9.26 = 4.49 J = 4.5 J

3. Angular momentum is conserved. As the size decreases, the moment of inertia will decrease as it depends on R. Since $I_1\omega_1 = I_2\omega_2$, in order to conserve angular momentum, the angular speed must be increased to compensate.

4. $\omega_1 = 20$ rads^{-1}, $\omega_2 = 18$ rads^{-1}, $m_{ADDED} = 0.2$ kg, r = 0.24 m

Moment of inertia of the object = $mr^2 = 0.2 \times 0.24^2 = 0.01152$ kgm^2

As angular momentum is conserved, $I_1\omega_1 = (I_1 + I_2)\omega_2$

$$I_1 \times 20 = (I_1 + 0.01152) \times 18$$

$$20I_1 = 18I_1 + 0.20736$$

$$I_1 = \frac{0.20736}{20-18}$$

$$= 0.10368$$

$$= 0.10 \text{ kgm}^2$$