TURNING POINTS

3-2 Einstein's theory of special relativity

1. (a) An inertial frame of reference is one which moves at constant velocity relative to another inertial frame of reference.
   (b) (i) A passenger jet moving at a constant velocity forms an inertial frame of reference for an object released at rest which remains at rest within it (within the passenger jet) according to Newton's 1st law.

   (ii) An accelerating passenger jet is a non-inertial frame of reference. An object released within it does not remain at rest.

   (c) (i) 'invariant' means that it has a fixed value and does depend on the motion of the light source or the motion of any observer.

   (ii) Einstein's other postulate was that in all inertial frames of reference, the laws of physics were the same.

2. $v = 0.98c$, $l_0 = 200m$, $c = 3.00 \times 10^8\ ms^{-1}$

   (a) (i) $t = \dfrac{200}{0.98 \times 3.00 \times 10^8} = 6.80 \times 10^{-7}\ s$ or 680 ns

   (ii) $t = \dfrac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ therefore $t_0 = t\sqrt{\left(1-\frac{v^2}{c^2}\right)}$

   $$= t\sqrt{\left(1-\frac{(0.98c)^2}{c^2}\right)}$$

   $$= 6.80 \times 10^{-7}\sqrt{(1-0.98^2)}$$

   $= 1.35 \times 10^{-7}\ s$ or 135 ns

   (b) $l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)} = 200\sqrt{(1-0.98^2)} = 40\ m$

   (Particles have dilated time and contracted distance).

3. (a) $v = 0.95c$, $m_0 = 9.11 \times 10^{-31}\ kg$

   relativistic mass, $m = \dfrac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \dfrac{9.11 \times 10^{-31}}{\sqrt{(1-0.95^2)}} = 2.917... \times 10^{-30}\ kg$

   (b) k.e. at this speed $= mc^2 - m_0c^2 = \dfrac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}c^2 - m_0c^2$

   $$= m_0c^2\left(\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} - 1\right)$$

   $$= 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 \times \left(\frac{1}{\sqrt{(1-0.95^2)}} - 1\right)$$

$= 9.11 \times 10^{-31} \times 9.00 \times 10^{16} \times 2.2$

$= 1.805... \times 10^{-13}$ J

$= 1.8 \times 10^{-13}$ J to 2 sf

(c) Accelerating p.d. = $\frac{1.8 \times 10^{-13} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$

$= 1.128... \times 10^{6}$ V

$= 1.13 \times 10^{6}$ V or 1.13 MV

4. (a) For m to be $100m_0$:

$$\frac{m_0}{m} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \text{ therefore } \frac{m_0}{100m_0} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\frac{1}{100} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\frac{1}{100^2} = \left(1 - \frac{v^2}{c^2}\right)$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{100^2} = 0.9999...$$

$\frac{v}{c}$ = 0.99994998…… = 0.99995 so v = 0.99995c

(b) Kinetic energy depends on both mass and speed. As the speed of the object approaches c its mass approaches infinity. The kinetic energy increase therefore comes from the mass approaching infinity even though the speed can't exceed c.