Chem Factsheet

www.curriculumpress.co.uk Number 66

Maths for Chemists 2

This Factsheet covers logarithms and their use in Chemistry.

What are logarithms?
Logarithms are closely related to powers, so before continuing, make sure you know and understand the power facts in the box (right).

To see how logarithms work, consider the following examples:

$100 = 10^2$ $\quad \log_{10}100 = 2$
$0.001 = 10^{-3}$ $\quad \log_{10}0.001 = -3$

So when we say "what is $\log_{10}$ 1000?", we are asking "What power must I raise 10 to, to get the answer 1000?" The answer is 3 - so $\log_{10}1000 = 3$

Although you can have logarithms to different bases, in Chemistry we will only be looking at logarithms to the base 10 ($\log_{10}$). To save time, we'll just write "log" when we mean "$\log_{10}$"

Calculating logs
It's easy to work out logs for numbers that are exact powers of 10 and it's worth being able to do so mentally to save time in the exam and guard against calculator error. But logs for other numbers have to be worked out on your calculator.

The button for finding log to the base 10 may be labelled:
LOG $\quad$ LOG_{10} $\quad$ LG

Note: do NOT use the button labelled ln or $\log_e$. This gives you logarithms to a different base - and hence the wrong answers!

Some calculators require you to put the number in first, then press the log button - in others, you press the log button first, then the number. Check which yours is by finding log100 (the answer should be 2).

Finding the number that has a particular log
There are two ways of doing this on your calculator:

- Use the x^y (or y^x) button to find 10 to the power of the number
eg: logx = 0.123. To find x, enter 10 x^y 0.123
- Use the INV or 2ND or SHIFT button, together with the LOG button
On some calculators you have to press INV LOG first, then the number; on others you press the number then INV LOG. Check which yours is by using this method to solve logx = -1.23 *(ans 0.0589)*

***Exam Hint**: You should **never** get a negative number from either of the above methods. It's impossible to find the log of a negative number - if you think you have, then you've probably pressed LOG not INV LOG.*

If a number has a whole-number log (eg logx = -4 or logy = 3) then you can find the number without a calculator:
logx = -4 so x = 10^{-4} = 0.0001 $\quad$ logy = 3 so y = 10^3 = 1000

Checking answers
Once you are confident with logs and can do whole-number examples in your head, you can use this method to check if your answer is plausible:

If logx = 1.57, since 1.57 is between 1 and 2, x is between 10^1 and 10^2
So if, when you found x, it wasn't between 10 and 100 - you'd know there was a mistake.

Powers

- Raising a number to a **positive** power means multiplying it by itself that number of times - eg $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$
- Special cases: any number to the power 1 is the number itself; any number to the 0 gives 1 - eg $2^1 = 2$ $\quad 3^0 = 1$
- Negative powers of a number are one over the corresponding postive power - eg $10^{-3} = \frac{1}{10^3} = \frac{1}{10\times10\times10} = \frac{1}{1000}$
- You can also have powers that are decimals - eg $10^{2.156}$
- You find powers on your calculator using the button marked x^y (or y^x) - eg to find $10^{2.156}$, type in 10, then x^y , then 2.156
(check - the answer should be 143.2)

When doing work on logarithms in Chemistry, you mainly deal with powers of 10 - so, a few facts about them:

- Positive powers of 10 are easy to recognise - just count the zeroes!
eg $10^1 = 10$ $\quad 10^3 = 1000$
- Negative powers of 10 are easy to recognise as decimals - just count the decimal places! eg $10^{-1} = 0.1$ $\quad 10^{-4} = 0.0001$
- Positive powers are larger than 1; negative powers smaller than 1

***Exam Hint**: Remembering these points can help you spot a wrong answer from keying into your calculator wrongly - and hence save a few marks!*

Before moving on to applying logs, check you understand the work so far by doing these questions. Try not to use your calculator in questions 1 and 3, and in questions 2 and 4, work out mentally the values your answer lies between in order to check your calculation,

1. Find:
a) log 100 $\quad$ b) log 1 $\quad$ c) log 10 $\quad$ d) log 0.1 $\quad$ e) log 0.001

2. Find:
a) log2 $\quad$ b) log 91 $\quad$ c) log0.61 $\quad$ d) log 0.003 $\quad$ e) log½

3. Find the number whose log is:
a) -2 $\quad$ b) 4 $\quad$ c) 1 $\quad$ d) 0 $\quad$ e) 6

4. Find the number whose log is:
a) 0.674 $\quad$ b) 2.18 $\quad$ c) -2.16 $\quad$ d) -0.0521 $\quad$ e) -1.32

Answers
1 a) 2 $\quad$ b) 0 $\quad$ c) 1 $\quad$ d) -1 $\quad$ e) -3
2 a) 0.301 $\quad$ b) 1.96 $\quad$ c) -0.215 $\quad$ d) -2.52 $\quad$ e) -0.301
3. a) 0.001 $\quad$ b) 10000 $\quad$ c) 10 $\quad$ d) 1 $\quad$ e) 1000000
4. a) 4.72 $\quad$ b) 151 $\quad$ c) 0.00692 $\quad$ d) 0.887 $\quad$ e) 0.0479

Logarithms and graphs

Sometimes graphs are required for data that cover a very wide range of values – going, for example, from 100 to 100 000. One example of this is successive ionisation energies for one element.This presents a problem, since if the largest values are to fit on the graph, it will be very hard to plot the small ones accurately.

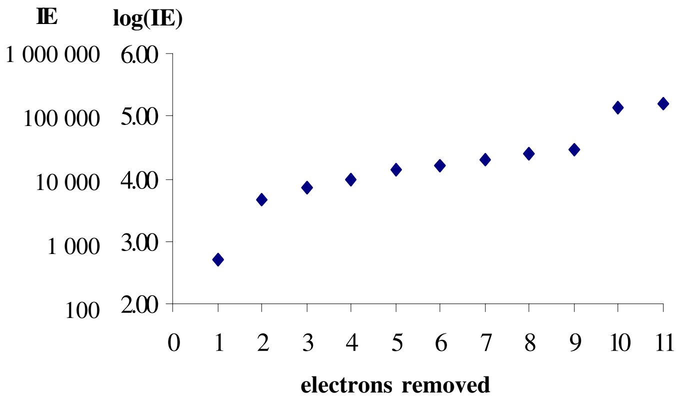
Logarithmic scales are used to overcome this problem. Instead of the actual value, the logarithm of the actual value is plotted. This can be done on one or both axes. This helps because the logarithms will not vary so much in size as the actual data values – eg log 100 = 2 and log100 000 = 5, so it would be easy to fit both of these on one graph. It is usual to label the axes with the original values instead of their logarithms; this means that the scale is likely to go up 1, 10, 100, 1000 etc.

Example: logarithmic graph of the successive ionisation energies of sodium.

Electrons removed	Energy required(kJ mol $^{-1}$)	log(energy)
1	490	2.69
2	4 560	3.66
3	6 940	3.84
4	9 540	3.98
5	13 499	4.13
6	16 600	4.22
7	20 100	4.30
8	25 500	4.41
9	28 900	4.46
10	141 000	5.15
11	158 700	5.20

We can see from the data that it would be hard to plot all the ionisation energies accurately, so it's sensible to use a logarithmic scale for them. There is no need to use one for the other axis, as these are small numbers.

The graph is shown below. Note the values of log(IE) on the axis are shown only for information here; the y-axis would normally just have IE values.



Logarithms and calculations on strong acids and bases

You need to be able to use logarithms in work on acids and bases.
The key definitions involving logs are:

$pH = -\log[H^+]$

$pOH = -\log[OH^-] \quad (= 14 - pH)$

Note that "p" here is representing "-log"

We will focus here on the aspects of the calculations involving logarithms; other Factsheets give further information on more general acid/base problems.

Example 1
Find the pH of a 0.1 mol dm^{-3} solution of hydrochloric acid

The key chemistry here is that as hydrochloric acid is a strong acid, it dissociates completely into ions - so $[H^+] = 0.1$

So $pH = -\log(0.1)$
$= -(-1) = 1$

Example 2
A solution of HBr is found to have a pH of 0.5. Find the concentration of the solution.

$pH = -\log[H^+]$
$0.5 = -\log[H^+]$
$\log[H^+] = -0.5$
$[H^+] = 10^{-0.5} = 0.316$ mol dm^{-3}

Since again we have a strong acid, this is also the concentration of the solution.

Example 3
Find the pH of a 2 mol dm^{-3} solution of sodium hydroxide

The key idea here is that as we are dealing with a base, we must use the two equations relating to pOH.
In addition, we know that sodium hydroxide is a strong base, and so will be fully dissociated into ions.

$pOH = -\log[OH^-]$
$= -\log 2$
$= -0.301$

$pH = 14 - pOH$
$= 14.301$

Example 4
A solution of potassium hydroxide has pH 13. Find its concentration

Again we use the two equations relating to pOH

$pOH = 14 - pH$
$= 14 - 13 = 1$

Now $pOH = -\log[OH^-]$
So $1 = -\log[OH^-]$
$-1 = \log[OH^-]$
$[OH^-] = 10^{-1} = 0.1$ mol dm^{-3}

Since we have a strong base, concentration of the solution is 0.1 mol dm^{-3}

Now try these examples. You can assume all acids and bases are strong.

1. Find the pH of a 0.5 mol dm^{-3} solution of nitric acid
2. Find the concentration of a solution of hydrochloric acid that has pH -0.6
3. Find the pH of a 3 mol dm^{-3} solution of potassium hydroxide
4. A solution of sodium hydroxide has pH 14. Find its concentration.

Answers
1. 0.301
2. 3.98 mol dm^{-3}
3. pOH = -0.477 so pH =14.477
4. pOH = 0 so 1 mol dm^{-3}

Weak acids & bases

When weak acids are considered, we must also take into account the dissociation constant K_a

$$K_a \text{ for the acid HA} = \frac{[H^+][A^-]}{[HA]} \qquad pK_a = -\log K_a$$

pK_a is often used as a measure of acid strength; unlike *pH* , it does not depend on the concentration of the solution.

Example: hydrofluoric acid has $K_a = 6.7 \times 10^{-4}$. Find pK_a

$pK_a = -\log (6.7 \times 10^{-4}) = 3.17$

Similarly for bases, we have the base dissociation constant K_b

$$K_b \text{ for the base BOH} = \frac{[B^+][OH^-]}{[BOH]} \quad pK_b = -\log K_b$$

Example: ammonia has $pK_b = 4.74$. Find K_b

$4.74 = -\log K_b$
$-4.74 = \log K_b$
$K_b = 1.8 \times 10^{-5}$

***Exam Hint**: A common student error is to treat a pK_a (or pK_b) value as if it were K_a (or K_b), and use it directly in the dissociation equation.*

pH of weak acids

To calculate the pH of a weak acid, you must first calculate $[H^+]$. To do this, you use the acid dissociation expression and the following assumptions:

- $[H^+] = [A^-]$, so the top of the fraction becomes $[H^+]^2$
  This is effectively assuming there are no other sources of either ion in the solution
- [HA] = original concentration of acid
  This is assuming an extremely small degree of dissociation - so that the concentration of undissociated acid is almost the same as the initial concentration.

Example. Calculate the pH of a 0.5 mol dm^{-3} solution of ethanoic acid ($K_a = 1.78 \times 10^{-5}$)

Step 1:

$$1.78\times 10^{-5} = \frac{[H^+][CH_3COO^-]}{[CH_3COOH]}$$

$$1.78\times 10^{-5} = \frac{[H^+]^2}{0.5}$$

Step 2: Rearrange the equation

$(1.78 \times 10^{-5}) \times 0.5 = [H^+]^2$

Step 3: Square root to find $[H^+]$

$$[H^+] = \sqrt{1.78\times 10^{-5} \times 0.5} = 0.00298$$

Step 4: Find pH

$pH = -\log[H^+] = 2.53$

Now try these examples

1. Nitrous acid (HNO_2) has pK_a =3.35
   a) Find K_a
   b) Find the pH of a 0.5 mol dm^{-3} solution of nitrous acid
2. Methanamine has pK_b =3.36
   a) Find K_b
   0.1 mole of methanamine is dissolved in 1 dm^3 of water.
   b) Assuming the degree of dissociation of methanamine is very low, and there are no other significant sources of OH^-, find $[OH^-]$
   c) Find the pH of this solution

1. a) 4.47×10^{-4} b) 1.8 2. a) 4.37×10^{-4}
2. b) Same approach as for an acid: $[OH^-]^2/0.1 = K_b$ This gives $[OH^-] = 6.61 \times 10^{-3}$
2. c) pH = 14 - pOH. pOH = 2.2 so pH = 11.8

Buffer Solutions

A buffer solution is a mixture of a weak acid and its salt (or less commonly, a weak base and its salt). The pH of a buffer solution changes only very little if strong acid or base is added to it.

There are a variety of examination questions on buffers - see Factsheet 64 - but here we will concentrate on those aspects related to logarithms. The key equation for use with buffer solutions is:

$$pH = pK_a - \log \frac{[HA]}{[A^-]}$$

Example 1: A buffer solution is made by dissolving 0.1 moles ethanoic acid and 0.2 moles sodium ethanoate in 1 dm^3 distilled water. Find its pH

(K_a for ethanoic acid = 1.78×10^{-5})

The key chemistry here is that we can assume the acid is effectively undissociated (it would dissociate very little anyway, and dissociation is further supressed by the presence of ethanoate ions from the salt). This means we can assume:

- *$[CH_3COO^-]$ is the original concentration of the salt*
- *$[CH_3COOH]$ is the original concentration of the acid*

We have: $[CH_3COO^-] = 0.2$ $[CH_3COOH] = 0.1$ $pK_a = -\log K_a = 4.75$

$pH = 4.75 - \log (0.1/0.2)$
$= 4.75 - \log 0.5$
$= 5.05$

***Exam Hint**: You may come across the equation for the buffer solution in slightly different formats. It's best to choose one format and stick to it - trying to remember more than one is likely to lead to mistakes.*

The second key fact needed for buffer solutions is:

If $[HA] = [A^-]$, then $pH = pK_a$

This relationship just comes from the fact that if $[HA] = [A^-]$, then $[HA]/[A^-] = 1$, and log1 = 0. So if you forget it, you can work it out again - but it's quicker to be able to remember it in an exam!

Example 2: A buffer solution is made by mixing 50cm^3 of each of 0.1 mol dm^{-3} methanoic acid and 0.1 mol dm^{-3} potassium methanoate. Its pH is 3.74. Find the acid dissociation constant for methanoic acid.

The acid and salt have equal concentrations, so $pH = pK_a$
So $pK_a = 3.74$
$\log K_a = -3.74$
$K_a = 1.82 \times 10^{-4}$

Note that if you hadn't noticed that the acid and salt had the same concentrations, you would have needed to find the concentration of each (0.05 for each, since the original solutions were diluted by a factor of 2) and then put the numbers into the equation.

Now try these examples

1. A acidic buffer solution contains equal concentrations of an acid,HA, and its sodium salt. Its pH is 4.61. Find K_a for HA
2. A buffer solution is 0.2 mol dm^{-3} with respect to ethanoic acid and 0.3 mol dm^{-3} with respect to sodium ethanoate. Find its pH (K_a for ethanoic acid = 1.78×10^{-5})
3. A buffer solution is 0.3 mol dm^{-3} with respect to acid HA. It also contains the sodium salt of the acid. The pH of the solution is 6.2 Find the concentration of the sodium salt in the solution (K_a for HA = 1.1×10^{-7}).

1. 2.45×10^{-5} 2. 4.9
3.First find log([acid]/[salt]) = 0.76 Then find [acid]/[salt] =5.7
So [salt] = [acid]/5.7 = 0.05 mol dm^{-3}

Acknowledgements:** This Factsheet was researched and written by Cath Brown* ***Curriculum Press, Bank House, 105 King Street, Wellington, TF1 1NU
***ChemistryFactsheets** may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber.*
No part of these Factsheets may be reproduced, stored in a retrieval system, or transmitted, in any other form or by any other means, without the prior permission of the publisher. ***ISSN 1351-5136***