Chem Factsheet

www.curriculum-press.co.uk Number 95

Rearranging Formulae

Two methods are given for rearranging formulae. The first method shows mathematically how the rearrangement is done; the second method, sometimes called 'cross multiplying', is a short cut. You should study and practise both methods but you may find it easier to use the method of cross multiplying for rearranging chemical formulae quickly.

Mole calculations

Perhaps the most important formula which is met in the early stages of a chemistry course is the one which relates the number of moles (n) of a substance to the mass of the substance (m) and the mass of one mole of substance (M), (the atomic or molecular mass expressed in grams). The formula is

$$n=m/M \qquad \text{(i)}$$

Example 1. Suppose we want to find the number of moles in 9 g of water. The molecular mass of water is (2 x 1 + 16) =18. To find the number of moles of water we simply substitute in the equation using m = 9 and M = 18 to give

$n=9/18$
$n=0.5$ mols

i.e. the number of moles in 9 g of water is 0.5 moles

Example 2. Suppose, however, we need to use the formula to calculate the mass of 3 moles of sulphur atoms. The atomic mass of sulphur is 32 so M = 32. We must now rearrange the formula to give us m as the subject of the formula.

1. Swap the two sides of the equation so that the term we want, m, is on the left hand side of the equation

   $m/M=n$

2. M is *dividing* on the left hand side of the equation. To 'get rid of ' the M on this side carry out the *opposite* mathematical operation. That is *multiply* both sides of the equation by M

   $M \times m/M = M \times n$ (because both sides of an equation are equal, if you do the same thing to both sides of the equation they remain equal)

3. Cancelling the M's on the left hand side gives:
   $m = M \times n$ which is the rearranged formula that we require.

   We now put in the quantities we are given:

   $m = 32 \times 3$
   $= 96$ g

   i.e. the mass of 3 moles of sulphur atoms is 96 g.

Cross multiplying

Another way to rearrange the equation is by 'cross multiplying'.

1. Swap the two sides of the equation so that the term containing what we want, m, is on the left hand side of the equation

   $m/M=n$

2. We need to 'get rid of' M on the left hand side of the equation to leave us with m. Take the M which is on the *bottom* of the fraction on the *left hand side* of the equation and move it across to the the *top* of the fraction on the *right hand side* of the equation.

   $$\frac{m}{M} = n$$

   $m = nM$

> *The rule for cross multiplying: if it is 'divide' on one side it becomes 'multiply' when you swap it to the other side of the equation and vice versa.*

Example 3. Rearrange formula (i) so that M is the subject of the formula. Hence calculate the molecular mass of a substance, 2 moles of which have a mass of 48 g.

$n=m/M$

1. Multiply both sides of the equation by M

   $n \times M = M \times m/M$

2. Cancel the Ms on the right hand side of the equation

   $n \times M = \cancel{M} \times m/\cancel{M}$
   $n \times M = m$

   This gives us M on the left hand side but we now have to get rid of n on the left hand side.

3. Since n is *multiplied* we do the opposite and *divide* both sides of the equation by n
   $n \times M = m$
   $n \text{ x } M/n = m/n$

4. Cancel the n's on the left hand side of the equation
   $\cancel{n} \text{ x } M/\cancel{n} = m/n$
   $M = m/n$

   This is the equation we want and so we can substitute the values we are given:

   $M = 48/2 =24$

   i.e. the molecular mass of the substance is 24.

Cross multiplying

It is quicker to rearrange the equation by cross multiplying.

$n=m/M$

1. Move the M on the bottom of the right hand side fraction to the top of the left hand side fraction.

$$n = \frac{m}{M}$$

$$Mn = m$$

2. Move the n on the top of the left hand side to the bottom of the right hand side fraction

   $M = m/n$ This gives the equation we need.

Questions

The number of moles of a solute in a solution is give by the formula:

$$n = \frac{C \times V}{1000}$$

where n is the number of moles, C is the concentration of the solution (in mol dm^{-3}) and V is the volume of the solution (in cm^3)

1. Rearrange the equation to make C the subject of the formula. Hence find the concentration of a solution of sodium hydroxide which contains 2 moles of solute in 500 cm^3 of solution.
2. Rearrange the equation to make V the subject of the formula. Hence find the volume (in cm^3) of a 3M solution of hydrochloric acid which contains 0.3 moles of solute.

Gas law calculations

Another important equation which often needs to be rearranged is the General Gas Equation,

$$pV=nRT$$

This equation can be rearranged to give each of the terms, p, V, n, R and T. The methods are exactly the same as those used above.

Example 4. Rearrange the general gas equation to make V the subject of the formula. $pV=nRT$

The term containing V is already on the left hand side of the formula. V is *multiplied* by p so, to 'get rid of' p, *divide* both sides of the equation by p

$$\frac{pV}{p} = \frac{nRT}{p}$$

Then cancel the p's on the left hand side of the equation to give

$$V = \frac{nRT}{p}$$

Or using the ***cross multiplying*** method,
Move the p on the left hand side of the equation to the bottom of the fraction on the right hand side

$$pV=nRT$$

$$V = \frac{nRT}{p}$$

Example 4. Rearrange the general gas equation to make T the subject of the formula, $pV= nRT$

Swap the two sides of the equation over so that the left hand side contains the term we want, T

$nRT = pV$

T is *multiplied* by n and R so *divide* both sides by nR to 'get rid of' n and R on the left hand side

$$\frac{nRT}{nR} = \frac{pV}{nR}$$

Cancel the n' and R's on the left hand side

$$\frac{\cancel{n}\cancel{R}T}{\cancel{n}\cancel{R}} = \frac{pV}{nR} \quad T=\frac{pV}{nR}$$

Using the ***cross multiplying*** method,

$pV= nRT$

Swap the two sides of the equation over so that the left hand side contains the term we want, T

$nRT = pV$

Move the n and R from left hand side to the bottom of the right hand side fraction.

$$T=\frac{pV}{nR}$$

Questions

3. Rearrange the equation $pV=nRT$ to make (a) p, (b) n and (c) R the subjects of the formula.
4. Calculate the number of moles in 0.1 m^3 of oxygen at a pressure of 90955 Pa and a temperature of 547 K
   ($R = 8.314\ J\ mol^{-1}\ K^{-1}$)

Energy calculations

When we use Hess's Law to work out energy changes we often end up with an equation which needs rearranging to give us the answer.

Example 5

In a calculation to find the enthalpy of formation of methane we might end up with the following equation:

$$-393 + 2(-286) = -890 + \Delta H_f[CH_4(g)]$$

(where $\Delta H_f[CH_4(g)]$ is the enthalpy of formation of methane)

We need to rearrange the equation to make the enthalpy of formation of methane, $\Delta H_f[CH_4(g)]$, the subject of the equation.

1. Swap the two sides of the equation over so that the term we need, $\Delta H_f[CH_4(g)]$, is on the left hand side of the equation.

$$-890 + \Delta H_f[CH_4(g)] = -393 + 2 \times (-286)$$

2. 'Get rid of' the term -890 by adding +890 to both sides of the equation. (As before, if you do the same thing to both sides of the equation they remain equal)

$$-890 + \Delta H_f[CH_4(g)] \mathbf{+890} = -393 + 2 \times (-286) \mathbf{+ 890}$$

3. The -890 and the +890 on the left hand side cancel each other out leaving the equation:

   $\Delta H_f[CH_4(g)] = -393 + 2 \times (-286) + 890$

   And this equation can be worked out to give:

   $\Delta H_f[CH_4(g)] = -393 + -572 + 890$

   $\Delta H_f[CH_4(g)] = -75 \text{ kJ mol}^{-1}$

A Short Cut

> If we move a term from one side of the equation to the other we change its sign. Starting with the same equation:

$-393 + 2 \text{ x } (-286) = -890 + \Delta H_f[CH_4(g)]$

1. Again swap the two sides of the equation so that the term we require is on the left hand side:

   $-890 + \Delta H_f[CH_4(g)] = -393 + 2 \text{ x } (-286)$

2. Move the -890 from the left hand side of the equation to the right and *change its sign:*

   $\Delta H_f[CH_4(g)] = -393 + 2 \times (-286) + 890$

3. Work out this equation as before to give:

   $\Delta H_f[CH_4(g)] = -393 + -572 + 890$

   $\Delta H_f[CH_4(g)] = -75 \text{ kJ mol}^{-1}$

Questions

5. Rearrange the following equations and find the value of the unknowns x, y and z:
   (a) $248 - 376 = x + 202$
   (b) $y + 123 = -478 + 1020$
   (c) $102 + 2(-346) = -135 - z$

More complex rearrangements

The same principles as shown above can be used to rearrange more involved equations

Example 6

Find the value of ΔH from the following equation:

$124 - 376 = 246 + 3\Delta H$

1. Swap over the two sides of the equation so that the unknown is on the left hand side.

   $246 + 3\,\Delta H = 124 - 376$

2. Move the 246 to the right hand side and change its sign

3 $\Delta H = 124 - 376 - 246$

4. Work out the right hand side

   $3\Delta H = 124 - 376 - 246$
   $= -498$

5. ΔH is *multiplied* by 3, so to obtain ΔH *divide* both sides by 3
   $\Delta H = -498/3$
   $\Delta H = -166$

Example 7

If an expression contains brackets these should be worked out before the addition and subtraction.

Make x the subject of the equation:

$$p = \frac{10\,(1-3x)}{7}$$

(Equations similar to this may arise in calculations involving partial pressures)

1. Cross multiply to put the 7 on the left hand side of the equation
   $7p = 10(1 - 3x)$
2. Multiply out the brackets
   $7p = 10 - 30x$
3. Put all the terms containing x on the left hand side and all the terms which do not contain x on the right hand side. *Change the signs where necessary.*
   $30x = 10 - 7p$

   Cross multiply to put the 30 on the right hand side

   $$x = \frac{10-7p}{30}$$

Example 8

Equations with roots or powers can be rearranged using the same principles

Rearrange the following equation to make $[H^+]$ the subject of the formula:

$$\frac{[H^+]^2}{C} = K_a$$

(This equation usually arises in second year modules. It relates the hydrogen ion concentration, $[H^+]$, to the concentration of a weak acid, C, and the acid dissociation constant, K_a)

Move the C over to the right hand side of the equation (*bottom to top!*) so that the squared term is by itself)

$[H^+]^2 = K_a \times C$

Since *square rooting* is the inverse of *squaring*, take the square root of both sides of the equation (As before, if you do the same thing to both sides of the equation they remain equal)

$[H^+] = \sqrt{(K_a \times C)}$

Questions

6. Make T the subject of the formula: $\Delta G = \Delta H - T\,\Delta S$
7. Make the concentration of NO_2, $[NO_2]$, the subject of the formula:
   $$K_p = \frac{[NO_2]^2}{[N_2O_4]}$$
8. Rearrange the following to make x the subject of the formula
   $$K = \frac{(3 - x)}{8}$$
9. Make k the subject of the formula: $R = k[NOCl]^2$
10. Make $p(H_2)$ the subject of the equation
    $$K_p = \frac{p(NH_3)^2}{p(H_2)^3 p(N_2)}$$

Answers to questions

1. $C = 1000 \times n/V$ 4 mol dm^{-3}
2. $V = 1000 \times n/C$ 100 cm^3
3. (a) $p = nRT/V$ (b) $n = pV/RT$ (c) $R = pV/nT$
4. 2 mol
5. (a) $x = 248 - 376 - 202 = -330$
   (b) $y = -478 + 1020 - 123 = +419$
   (c) $z = -102 + 2 \times 346 - 135 = +455$
6. $T = \dfrac{\Delta H - \Delta G}{\Delta S}$
7. $[NO_2] = \sqrt{(K_a \times [N_2O_4])}$
8. $x = 3 - 8K$
9. $k = R/[NOCl]^2$
10. $p(H_2) = \sqrt[3]{p(NH_3)^2/(K_p \times p(N_2))}$

Acknowledgements:** This Factsheet was researched and written by Chris Senior* ***Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU. ISSN 1351-5136