Chem Factsheet

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Logs and Powers in Chemistry

Logs (short for logarithms) and powers are closely related. Logs arise in chemistry in the calculation of pH and powers are encountered in expressions for equilibrium constants and rate equations. This Factsheet is concerned with explaining what logs and powers are and how to calculate with them. It is not a sheet on pH calculations or rate equations - these are dealt with elsewhere.

Powers of 10

Consider the following numbers:
100
1000
10000

These can of course be written as follows:

$100 = 10 \times 10$
$1000 = 10 \times 10 \times 10$
$10000 = 10 \times 10 \times 10 \times 10$

And these can be written as powers of 10:

$100 = 10 \times 10 = 10^2$
$1000 = 10 \times 10 \times 10 = 10^3$
$10000 = 10 \times 10 \times 10 \times 10 = 10^4$

where 10^2 means two tens multiplied together (10×10), 10^3 means three tens multiplied together ($10 \times 10 \times 10$) and so on.

We can extend this sequence of number upwards as far as we like by continuing to multiply by ten. Each time we multiply by ten we add one to the power, $100000 = 10^5$ and so on.

However, what happens if we extend the sequence in the other direction? If we go from 1000 to 100 we are dividing by 10 and the number in power form goes from 10^3 to 10^2. Each time we divide by 10 we subtract 1 from the power. So the sequence goes like this (going downwards):

Number	Number in Power Form
1000	$= 10^3$
100 $= (1000 \div 10)$	$= 10^2$
10 $= (100 \div 10)$	$= 10^1$
1 $= (10 \div 10)$	$= 10^0$
0.1 $= (1 \div 10)$	$= 10^{-1}$
0.01 $= (0.1 \div 10)$	$= 10^{-2}$
0.001 $= (0.01 \div 10)$	$= 10^{-3}$

There are several things we can see from this table.

1. 10^1 is just 10. This is true for any number - any number raised to the power of 1 is just itself.
2. 10^0 is just 1. Again this is true for any number. Any number to the power of 0 is 1.
3. 10 to the power of a negative number is 1 divided by 10 to the positive power (sometimes called the reciprocal). So

$10^{-1} = 1/10^1 = 1/10 = 0.1$
$10^{-2} = 1/10^2 = 1/100 = 0.01$
$10^{-3} = 1/10^3 = 1/1000 = 0.001$

Logarithms

> *The logarithm or log (to the base of 10) of a number is defined as the power to which we have to raise 10 to give us that number.*

If we look at the table of powers of 10:

Number (n)	Number in power form	$\log_{10}$ n
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	-1
0.01	10^{-2}	-2
0.001	10^{-3}	-3

The entry in the third column is simply the power of the number in the second column and it is the number in the third column which is called the log. 10 is referred to as the *base* of the log. We can use other numbers as the base but for working out pH calculations we always use base 10.

Question 1. (a) What is $\log_{10}$ of 10000? (b) What is $\log_{10}$ 0.00001?

So it is quite easy to find the log of a number to the base of 10 so long as the number is a whole number power of 10. What happens if we need to find the log of a number such as, say, 500 or 0.0027 which is between the numbers in the table above? It is actually quite difficult to work out these logs but fortunately it can be done on a scientific calculator.

For example, a calculator shows that the $\log_{10}$ of 500 is 2.699 (to 3 decimal places). If we were to put this number in the table it would be between 100 and 1000 and so the log would be between 2 and 3:

Number(n)	Number in power form	$\log_{10}$ n
1000	10^3	3
500	$10^{2.699}$	2.699
100	10^2	2

Similarly, a calculator shows that the $\log_{10}$ of 0.0027 is -2.569 (to 3 decimal places). Putting this number in the table would give:

Number(n)	Number in power form	$\log_{10}$ n
0.01	10^{-2}	−2
0.0027	$10^{-2.569}$	−2.569
0.001	10^{-3}	−3

> Notice that the logs of numbers less than 1 are always negative. Notice also that we cannot have a log of a negative number. If you try it on your calculator you will get an error meassage.

Using a calculator to find logs

Virtually all scientific calculators can be used to find logs. On modern calculators it is simply a matter of pressing the "log" key, typing in the number and pressing "=". (On older scientific calculators it was necessary to type in the number first and then press the log key. Check in the instructions for your calculator which way to do it.)

> NB Be sure to press the key marked "log" and not the key marked "ln". The "ln" key gives logs to a different base and would give the wrong answers in pH calculations.

Question 2 Use your calculator to find :
(a) $\log_{10}$ 12.345? (b) $\log_{10}$ 0.012345 (give your answers to 3 d.p.)

Inverse logs

In calculations on pH we often need to take the inverse log of a number. In other words, find a number whose log is given.

For example, find the inverse log of 0.301.

What this means is, "find the number whose log is 0.301", or to put it algebraically, "if $\log_{10} x = 0.3010$, what is x?"

From the definition of logs we know that the log of a number is the power to which we have to raise 10 to give us that number. So if the log of the number is 0.301 then the number itself must be $10^{0.301}$.

We can again find inverse logs on a scientific calculator. On modern calculators press the "shift" key and then the "log" key. (It is usually marked as 10^x). Then enter the number and press "=".

In this case, the inverse log of 0.301 is $10^{0.301}$ = 1.9999 (= 2.00 to 3 significant figures).

In general if $n = \log_{10} x$, then $x = 10^n$. Inverse logs are also sometimes called antilogs.

Question 3. Using your calculator:
(a) What is the inverse log of 4?
(b) What is the antilog of 2.35
(c) What is the number whose log is -2.5?
(d) Calculate $10^{0.35}$
(e) If $\log_{10} x = -1.65$ what is x?
(Give your answers to 3 significant figures)

Using logs in pH calculations

pH is a measure of the concentration of hydrogen ions in a solution. Logs are used in pH calculations to make the numbers used for small concentrations of hydrogen ions easier to handle. Since most (though not all) of the concentrations of hydrogen ions that we come across in chemistry are less than one, the log of the hydrogen ion concentration would be a negative number. So, again, to give us easier numbers, the pH is taken to be the *negative* log of the hydrogen ion concentration.

> *pH is defined as the negative logarithm to the base 10 of the hydrogen ion concentration* or in symbols:
>
> $$pH = -\log_{10}[H^+]$$
>
> *where $[H^+]$ is the hydrogen ion concentration of a solution in mol dm^{-3}*

Example (1): Calculate the pH of a solution of hydrochloric acid whose hydrogen ion concentration is 0.010 mol dm^{-3}.

Answer:
The concentration of hydrogen ions is represented as $[H^+]$

$[H^+] = 0.010$
$pH = -\log_{10}[H^+]$
$= -\log_{10} 0.010$
$= 2.00$

(using a calculator or using the table on previous page. Don't forget to change the sign.)

Example (2): Calculate the pH of a solution of nitric acid whose hydrogen ion concentration is 0.0050 mol dm^{-3}.

Answer: $[H^+] = 0.0050$
$pH = -\log_{10}[H^+]$
$= -\log_{10} 0.0050$
$= 2.30$ (using a calculator)

Example (3): Calculate the pH of a solution of sulphuric acid whose hydrogen ion concentration is 1.00 mol dm^{-3}.

Answer: $[H^+] = 1.00$
$pH = -\log_{10}[H^+]$
$= -\log_{10} 1.00$
$= 0.00$

Example (4): Calculate the pH of a solution of sodium hydroxide whose hydrogen ion concentration is 3.00×10^{-11} mol dm^{-3}.

Answer: $[H^+] = 3.00 \times 10^{-11}$
$pH = -\log_{10}[H^+]$
$= -\log_{10} (3.00 \times 10^{-11})$ (enter as (-)/log/3/EXP/(-)/11. You don't need to put the 10 in – the calculator does it for you)
$= 10.52$

Example (5): A solution has a pH of 2.50. What is the concentration of hydrogen ions in the solution?

Answer: To do this we need to find the inverse log of -2.50

$pH = 2.50$
$-\log_{10}[H^+] = 2.50$ (from the definition of pH)
$\log_{10}[H^+] = -2.50$ (change the signs)
$[H^+] = 10^{-2..5}$ (don't forget the minus sign)
$[H^+] = 3.16 \times 10^{-3}$ (find the inverse log using a calculator)

i.e. the hydrogen ion concentration of a solution whose pH is 2.5 is 3.16×10^{-3} mol dm^{-3}

Example (6): A solution has a pH of 7.00. What is the concentration of hydrogen ions in the solution?

Answer: $pH = 7.00$
$[H^+] = 1.00 \times 10^{-7}$

i.e. the hydrogen ion concentration of a solution whose pH is 7 is 1.00×10^{-7} mol dm^{-3}

Example (7): A solution has a pH of 12.30. What is the concentration of hydrogen ions in the solution?

Answer: $pH = 12.30$
$[H^+] = 10^{-12.5}$
$[H^+] = 5.01 \times 10^{-13}$

i.e. the hydrogen ion concentration of a solution whose pH is 12.30 is 5.01×10^{-13} mol dm^{-3}

Logs to other bases

Although pH calculations always use 10 as the base for logs, in other areas of chemistry logs to the base e are used. (e is a number which has some special mathematical properties and has a value of approximately 2.718.) These logs are sometimes called natural logs and are often given the symbol ln or $\log_e$. Although you may see these logs in your reading and on calculators, you will not be required to use them in current A level examinations.

Powers

When the same number is multiplied together several times we can express this by writing it as a power.

For example,
$2 \times 2 = 2^2 = 4$,
$10 \times 10 \times 10 \times 10 = 10^4 = 10000$,
$1.5 \times 1.5 \times 1.5 = 1.5^3 = 3.375$

As we saw earlier any number raised to the power of 1 is itself. e.g. $3^1 = 3$
Any number raised to the power of 0 is 1. e.g. $5^0 = 1$
Any number raised to a negative power is 1 divided by (or the reciprocal of) the number raised to the positive power. e.g. $3^{-2} = 1/3^2$.

> Numbers expressed as powers can be **multiplied by adding their powers**.

e.g. $2^3 \times 2^2 = 2^5$ (adding the powers 2 + 3)

We can see that this is so because $2^3 = 2 \times 2 \times 2$ and $2^2 = 2 \times 2$ and so $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$

> Numbers expressed as powers can be **divided by subtracting their powers.**

e.g. $2^6 \div 2^4 = 2^2$

We can see that this is so because $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ and $2^4 = 2 \times 2 \times 2 \times 2$
and so $2^6 \div 2^4 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2 \times 2) = 2 \times 2 = 2^2$

NB $2^6 \div 2^4$ could also be written as $2^6 \times 1/(2^4)$ or $2^6 \times 2^{-4}$

These rules are also true for powers which are not whole numbers.
e.g. $4^{3.7} \div 4^{1.7} = 4^2$ (you would have to use a calculator to check this)

> Numbers expressed as powers can be **raised to a power by multiplying the powers together**.

e.g. $(2^2)^3 = 2^6$ (the powers 2 and 3 are multiplied to give 6)

We can see that this is so because $2^2 = 2 \times 2$ and $(2^2)^3 = (2^2) \times (2^2) \times (2^2)$
and so $(2^2)^3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$

> We can **take roots of a number expressed as a power by dividing the powers**.

e.g. $\sqrt{(2^6)} = 2^3$ (divide the power 6 by the root 2)

We can check this because $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ and the square root of 64 is 8 which is 2^3.

Roots can also be written as a number raised to a fraction so $\sqrt{10}$ could be written as $10^{1/2}$, the cube root of 8 could be written as $8^{1/3}$

Example (8) A reaction is first order with respect to a reactant P and first order with respect to a reactant Q. It has the rate equation rate = k[P][Q] The concentrations of P and Q are measured in mol dm^{-3} and the reaction rate is measured in mol dm^{-3} s^{-1}

(a) What is the overall order of the reaction? (b) What are the units for the rate constant?

Answer:
(a) The overall order of the reaction is the sum of the orders with respect to each of the reactants which in this case is 2.
(b) The units must be equal on each side of the rate equation.

mol dm^{-3} s^{-1} = units for k × mol dm^{-3} × mol dm^{-3}

The units for k are found by rearranging the equation to give

$$\text{units for k} = \frac{\text{mol dm}^{-3}\ \text{s}^{-1}}{(\text{mol dm}^{-3} \times \text{mol dm}^{-3})}$$

This can be simplified to give:

$$\text{units for k} = \frac{\text{mol dm}^{-3}\ \text{s}^{-1}}{(\text{mol}^2\ \text{dm}^{-6})} \quad \text{(multiply by adding powers)}$$

$$\text{units for k} = \text{mol}^{-1}\ \text{dm}^3\ \text{s}^{-1} \quad \text{(divide by subtracting powers)}$$

Finally, for reference, here is a summary of some important equations used for handling powers.

Multiplication of powers	$a^m \times a^n$	=	a^{m+n}
Division of powers	$a^m \div a^n$	=	a^{m-n}
Powers of powers	$(a^m)^n$	=	a^{mn}
Roots of powers	$\sqrt[n]{(a^m)}$	=	$a^{m/n}$
Also remember	a^0	=	1
	a^1	=	a
	a^{-n}	=	$1/a^n$

Answers to questions in text

1. (a) 4.00
   (b) -5.00

2. (a) 1.091
   (b) -1.909

3. (a) 10^4 or 10000
   (b) 224
   (c) 3.16×10^{-3}
   (d) 2.24
   (e) 0.0224

Practice Questions

1. Find the log to the base 10 to the following numbers. Give your answer to 3 decimal places.
   (a) 100
   (b) 0.01
   (c) 5
   (d) 0.05
   (e) 500
   (f) 0.00135
   (g) 10^{-3}
   (h) 2.35×10^{-5}
   (i) 2.35×10^{5}
   (k) 6.02×10^{-14}

2. Find the inverse log of the following. Give your answer to three significant figures.
   (a) 3
   (b) -2
   (c) 3.25
   (d) -2.73
   (e) 0

3. Calculate the pH of the following solutions. Give your answer to 2 decimal places.
   (a) A solution of hydrochloric acid whose hydrogen ion concentration 0.0010
   (b) A solution of nitric acid whose hydrogen ion concentration 0.0025
   (c) A solution of propanoic acid whose hydrogen ion concentration 1.20×10^{-3}
   (d) A solution of sodium hydroxide whose hydrogen ion concentration 1.00×10^{-13}
   (e) A solution of potassium hydroxide acid whose hydrogen ion concentration 2.35×10^{-14}

4. Calculate the hydrogen ion concentration of the following solutions. Give your answer to three significant figures.
   (a) A solution of hydrochloric acid whose pH is 2.00
   (b) A solution of ethanoic acid whose pH is 3.50
   (c) A solution of iron(II) sulphate whose pH is 6.10
   (d) A solution of sodium hydroxide whose pH is 14.00
   (e) A solution of ammonia whose pH is 11.50

5. Simplify the following expression:
   (a) mol $dm^{-3} \times$ mol dm^{-3}
   (b) mol^2 $dm^{-6} \times$ mol dm^{-3}
   (c) mol $dm^{-3} \div mol^2$ sm^{-6}
   (d) (mol $dm^{-3} \times mol^2$ dm^{-6}) $\div$ mol dm^{-3}
   (e) $(\text{mol dm}^{-3})^2$

Answers

1. (a) 2 (b) -2 (c) 0.699 (d) -1.301 (e) 2.699
   (f) -2.870 (g) -3 (h) -4.629 (i) 5.371 (k) -13.221

2. (a) 10^3 or 1000 (b) 10^{-2} or 0.01 (c) 1780 (d) 1.86×10^{-3} (e) 1

3. (a) 3.00 (b) 2.60 (c) 2.92 (d) 13.00 (e) 13.63

(If you use a calculator to work out 3(d) be sure to enter "log 1 EXP -13" and not "log 10 EXP -13". The calculator interprets 10 EXP -13 as $10 \times 10^{-13} = 10^{-12}$. But you shouldn't need a calculator for this anyway!)

4. (a) 1.00×10^{-2} (b) 3.16×10^{-4} (c) 7.94×10^{-7} (d) 1.00×10^{-14} (e) 3.16×10^{-12}

5. (a) mol^2 dm^{-6} (b) mol^3 dm^{-9} (c) mol^{-1} dm^{+3} (d) mol^2 dm^{-6} (e) mol^2 dm^{-6}

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