Chem Factsheet

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The Ideal Gas Equation

To succeed with this topic you need to know:-
- **How to rearrange formulae;**
- **How to do basic mole calculations involving masses.**

After working through this Factsheet you will understand:-
- **What the Ideal Gas Equation is;**
- **How it convert into the SI units needed for the Ideal Gas Equation;**
- **How to do calculations using the Ideal Gas Equation;**
- **When the Ideal Gas Equation is a good approximation to the behaviour of real gases.**

1. What is the Ideal Gas Equation?
The Ideal Gas Equation describes the link between the temperature, pressure and volume of a gas.

The Ideal Gas Equation is:
$pV = nRT$

Where: p = pressure in Pa (Nm^{-2})
V = volume in m^3
n = number of moles of gas
R = universal gas constant = 8.31 $JK^{-1}\,mol^{-1}$
T = temperature in Kelvin

Units and conversions
You might need to convert units of **volume** and **temperature** when using the Ideal Gas Equation.

Volume
To use the equation, you need volume in m^3. You are likely to start out with volume in cm^3 or litres (dm^3).
The key figure to remember is that 1 m^3 = 1 000 000 cm^3.
If you are not good at remembering this, here's how to work it out:-
1 m = 100 cm. So cubing everything: $1^3\,m^3 = 100^3\,cm^3$ = 1 000 000 cm^3.
(This also explains why 1 litre = 1 dm^3: 1 dm = 10cm, so 1dm^3 = $10^3\,cm^3$)
Since 1 litre (or dm^3) = 1000 cm^3, that means 1 m^3 = 1000 dm^3.

Once you know the relationship between cm^3/ dm^3/ m^3, you just need to remember whether to divide or multiply. To help you do this, think about 1 cm^3. This is clearly much smaller than 1 m^3, so when converting to m^3, you are wanting to get a smaller answer than you started with - so you have to **divide** when converting to m^3.

To convert cm^3 to m^3, ***divide by 1 000 000***
To convert litres (= dm^3) to m^3, ***divide by 1 000***

Temperature
The difference between Kelvin and Celsius temperatures is the temperature counted as zero. Zero Kelvin is known as **absolute zero** - it is the lowest temperature possible, the temperature at which particles stop moving. It is approximately -273°C.

To convert Celsius temperatures to Kelvin, add 273.

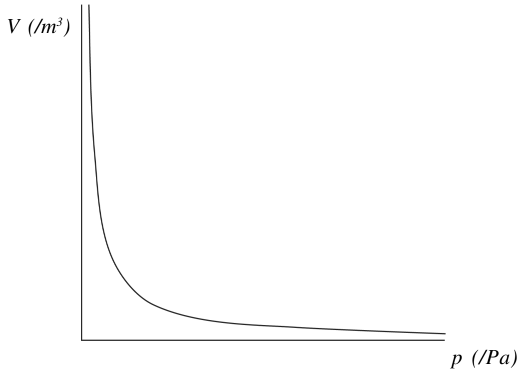
Pressure
The only conversion needed here is from **kilopascals** (kPa) to pascals - as with kilometres and metres, you just multiply by 1000.

2. Understanding the Ideal Gas Equation
Although you can just plug numbers into the equation to do calculations, it is useful to have an idea what this equation is telling us about the relationship between pressure, volume and temperature, because it helps both in understanding the concepts and appreciating when an answer is implausible.

First suppose the **temperature** of a gas remains constant. Then the relationship between pressure and volume can be summarised by

$$pV = constant$$

This leads to a graph of the type shown below:-

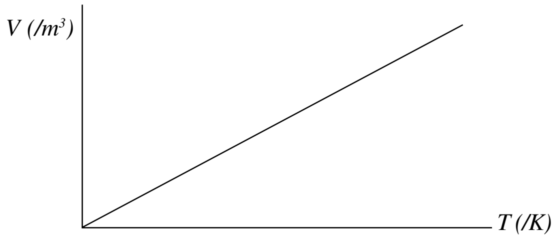


This should - hopefully - be fairly intuitive - if we put a lot of pressure on something (i.e. squash it) its volume will decrease - so high pressure gives low volume. More precisely, it means that if we double the pressure - keeping everything else unchanged - then the volume will halve. So if a particularly quantity of gas at a constant temperature had a volume of 1 m^3 at a pressure of 100 Pa, then its volume would be 0.5m^3 at a pressure of 200 Pa, or 2m^3 at a pressure of 50 Pa.

Now suppose instead that the **pressure** is kept constant. In this case, the equation becomes

$$V = constant \times T$$

This gives a straight-line graph:-



Again, this should (to some extent) be intuitive - you would expect things to expand when you heat them. However, it's important to note that the graph only looks exactly like this when temperature is measured in Kelvin.

For temperatures measured in Kelvin, because this graph goes through the origin, we can say that doubling the temperature results in doubling the volume, or halving the temperature in halving the volume. For temperatures measured on other scales (eg celsius, fahrenheit), although we still get a straight line graph, it does not pass through the origin. That means that we will **not** find the volume doubling if the temperature increases from 10°C to 20°C. This is why it is important to measure the temperature in Kelvin!

Now suppose the **volume** is kept constant. The equation now becomes:

$$p = \text{constant} \times T$$

This again gives a straight-line graph through the origin if the temperature is measured in Kelvin.

So this is telling us that doubling the Kelvin temperature results in the pressure doubling if the volume remains constant. It may not be obvious why increasing the temperature should change the pressure - to explain it, we need to consider the motion of the particles. When we increase the temperature of a gas, the particles in it move faster. This will mean they will hit the sides of the container more often, and harder. Since the pressure is caused by the force of the particles hitting the sides of the container, this means the pressure increases.

From a chemical point of view, it is also useful to think how (and why) n, the number of moles, will affect the other quantities.

- If p and T are constant, then an increase in n will result in an increase in V - another straight line graph. This is telling us that, all other things being equal, the more moles of gas we have, the larger volume it will occupy.
- Now instead if V and T are constant, then an increase in n will result in an increase in p. That's saying that if we have the same volume, and put more moles of gas in it, the pressure will be higher. This is because we will have more particles in the container, so they will hit the sides more often, increasing the pressure.
- Finally, suppose p and V are constant. Then here, n and T will be inversely proportional - the more moles of gas you have, the lower temperature they must be at. This probably seems counterintuitive! The explanation relates to the relationship between speed of movement of particles. If particles are at a lower temperature, they will hit the wall less often. So having more particles at a low temperature in a given container can result in the same pressure as fewer particles at a higher temperature. So to keep the pressure and volume the same, we must reduce the temperature if we increase the number of moles.

3. Calculations using the Ideal Gas Equation

Questions will typically not just give you three out of the four variables involved in the equation, and ask you to calculate the fourth. They commonly require you also to use your knowledge of the connection between moles, mass and M_r also - for example, giving you a mass of a known substance and requiring you to convert it to moles. The best way to see how this works is with examples.

Example 1. A gas cylinder, of volume 5 dm^3, contains 8 g of oxygen gas. Use the ideal gas equation to calculate the pressure of the oxygen gas in the cylinder at a temperature of 25°C. (The gas constant $R = 8.31\ JK^{-1}mol^{-1}$).

First write down which of the variables from the equation we are given, and convert to appropriate units:-

$V = 5\ dm^3 = 5 \div 1000 = 0.005\ m^3$. $T = 25^oC = 298\ K$

Note down which variable we want to find:-

We are asked for p

Work out what other variable from the equation we need to do this:-

We know R (it's a constant given to us), so we need to find n

Use the other information in the question to work this out:-

*We are told a **mass** of oxygen. We know the M_r for O_2 is 32.*
So we can find moles of oxygen = mass $\div M_r = 8 \div 32 = 0.25$

Put everything into the Ideal Gas Equation:-

$pV = nRT$ so $p \times 0.005 = 0.25 \times 8.31 \times 298$
$p = 0.25 \times 8.31 \times 298 \div 0.005 = 123819$ Pa $= 124000$ Pa (3 sig figs)

> ***Exam Hint: -*** *One of the commonest mistakes to make is to just substitute the numbers you are given in automatically, without checking what they are - for example, substituting a mass in as if it were a number of moles. Take the time to read the information carefully, and to note down exactly what you are being told.*

> ***Exam Hint: -*** *Another common mistake in examples like this one is to use the A_r rather than M_r - i.e. 16 rather than 32 for oxygen. If you always write the formula down of the gas you are dealing with, this should help you avoid this error.*

Example 2. Compound **X** is an oxide of nitrogen. At 380K, a gaseous sample of **X** of mass 0.229 g occupied a volume of 150cm^3 at a pressure of 105 kPa. Calculate the relative molecular mass of **X**

Write down what we are given from $pV = nRT$:

$V = 150\ cm^3 = 0.00015\ m^3$. $T = 380\ K$
$p = 105\ kPa = 105000\ Pa$

Since we are given p, V and T (and we know R) - there isn't any choice what to do next! We have to calculate n:-

$pV = nRT$ so $105000 \times 0.00015 = n \times 8.31 \times 380$
So $n = 105000 \times 0.00015 \div (8.31 \times 380) = 0.00499$

Now we must relate this to what the question has asked us - it wants a value for M_r.

We know that we have 0.00499 moles (from above) and the mass is 0.229 g. So using M_r = mass/ moles, we get $M_r = 46$ (2 sig figs)

If you have time, you could do a double check that this is plausible for an oxide of nitrogen - NO_2 has M_r of 46.

> ***Exam Hint: -*** *Exam questions typically ask you to state the Ideal Gas Equation first - this is an easy mark to get!*

Example 3. Copper (II) nitrate undergoes thermal decomposition as shown in the equation below.

$$2Cu(NO_3)_2(s) \rightarrow 2CuO\,(s) + 4NO_2\,(g) + O_2(g)$$

A sample of copper (II) nitrate was heated until it completely decomposed. The gases formed occupied a volume of 7750 cm^3 at 100°C and a pressure of 1.00×10^5 Pa. Find the mass of the original sample.

Write down what we are given from $pV = nRT$:

$V = 7750\ cm^3 = 0.00775\ m^3$. $T = 100^oC = 373\ K$
$p = 10^5\ Pa = 100000\ Pa$

As in the previous example, we have to calculate n

$pV = nRT$ so $100\,000 \times 0.00775 = n \times 8.31 \times 373$
So $n = 100\,000 \times 0.00775 \div (8.31 \times 373) = 0.250$

Now we need to relate this to what we are asked and what we are given. Since we are given the equation for the decomposition, we clearly have to use it. We are asked for the mass of the original sample - so we have to link this to our answer from the ideal gas equation.

We know the total moles of the two gases = 0.25
But there are 4 moles of NO_2 for every one of O_2
So there must be 0.2 moles of NO_2 and 0.05 moles of O_2

Now we need to link this to moles of $Cu(NO_3)_2$.
From the equation, moles of $Cu(NO_3)_2 = 2 \times$ moles of $O_2 = 0.1$ moles
So mass of $Cu(NO_3)_2$ = moles $\times M_r = 0.1 \times 187.5 = 18.8$ g (3 sig figs)

> ***Exam Hint: -*** *Take time to double check the **units** you 've been given, and are asked for. Wrong units means lost marks!*

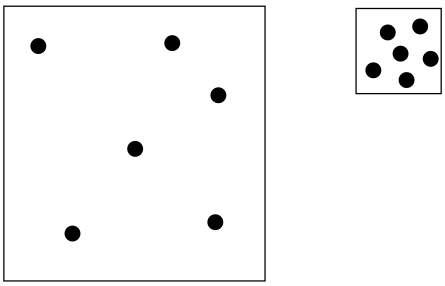
4. Validity of the Ideal Gas Equation for real gases

To understand the validity (or otherwise) of the Ideal Gas Equation, you need to understand the assumptions that are made when it is derived. These are:-

> *Assumptions made to derive Ideal Gas Equation*
> I *The gas consists of very small particles whose sizes are negligibly small compared to the distance between them*
> II *The particles in the gas are in constant random motion, so they collide with each other and the walls of the container*
> III *The collisions are perfectly elastic (like a pool or snooker ball, for example)- this means no energy is lost in the collisions and the particles never stick together*
> IV *The particles have no effect on each other except during collisions*
> V *The kinetic energy (motion energy) of the particles is proportional to the temperature (in Kelvin) of the gas.*

So a real gas will behave most like an idea one when these conditions are closest to being satisfied - and will deviate from the Ideal Gas Equation when the conditions fail.

Consider condition I first. This one works perfectly well at pressures close to "normal" - i.e. atmospheric pressure. However, if we have a high pressure, then the particles are much more crowded together. This means that the actual particles are filling a greater proportion of the space. The diagram below illustrates this; in the first case, the particles are a small proportion of the square, but in the second, they form an appreciable fraction of it.



The net result of this is that the real gas takes up slightly more volume than we'd expect at high pressures - because the volume of the actual particles starts to "count".

Now consider condition IV. This is saying that there are no forces between the particles in general - they only affect each other when colliding. We know this cannot be true - they will certainly experience Van der Waals forces, and for polar molecules there will be dipole-dipole interactions. This leads to the pressure of the gas on the container being slightly smaller than expected from the equation.

The gases for which this assumption is closest to being true are those for which these forces are as small as possible - eg noble gases, particularly helium and neon, as their atoms are small. Those it is least close to true for are gases such as hydrogen fluoride, which have hydrogen bonding.

How important these forces between the particles are also depends on the speed with which the particles are moving. If they are moving at high speeds, then they will hardly "notice" the intermolecular forces, while if they are moving only slowly, then the intermolecular forces become more significant. So since the speed of the particles is determined by the temperature, there will be significant deviations from ideal behaviour at low temperatures.

> *Gases most closely approximate "ideal" behaviour if:-*
> - *The temperature is high*
> - *The pressure is low*
> - *The gas is monatomic with a low M_r*

> *There are significant deviations from ideal behaviour if:-*
> - *The temperature is low*
> - *The pressure is high*
> - *The gas is polar and/or has a very high M_r*

Questions

1. State the ideal gas equation, explain the meaning of each of the variables in it and give their units.
2. A reaction produces 0.42 moles of gaseous products at a temperature of 700°C. These are contained in a sealed container of volume 2 dm^3. Calculate the pressure in the container.
3. In a sealed container of volume 0.005 m^3, 0.2 moles of carbon monoxide react completely with 0.1 moles of oxygen to produce carbon dioxide. After the reaction, the temperature in the container is 110°C. Find the pressure in the container.
4. A sample of hydrogen gas has mass 0.115 g. It occupies a volume of 3.5 dm^3 at a temperature T and a pressure of 100 kPa. Find the value of the temperature T.
5. A sample of oxygen is produced by the decomposition of hydrogen peroxide. The sample occupies 1.6 dm^3 at a pressure of 105 kPa and a temperature of 298 K. Find the mass of the sample.
6. A sample of gas has mass 0.1825 g and occupies a volume of 122 cm^3 at a temperature of 20°C and a pressure of 100 kPa. Calculate its relative molecular mass.
7. 0.32 g of a volatile liquid is heated to a temperature of 80°C in an evacuated container.; all the liquid has then vapourised. The container has volume 100 cm^3. The pressure in the container after the liquid has vapourised is 58.7 kPa. Find the relative molecular mass of the liquid.
8. (a) State the conditions in which a real gas most closely approximates the behaviour of an ideal gas
   (b) State, with reasons, which of the following are likely to show significant deviations from the Ideal Gas Equation:-

Gas	Pressure	Temperature
Hydrogen fluoride	100 kPa	473 K
Argon	100 kPa	423 K
Ammonia	5000 kPa	423 K
Xenon	10000 kPa	20 K

Answers

1 and 8 (a), see text.

2. $p \times 0.002 = 0.42 \times 8.31 \times 973$ so $p = 1700$ kPa (3 sig figs)
3. This produces 0.2 moles of carbon dioxide.
   $p \times 0.005 = 0.2 \times 8.31 \times 383$ so $p = 127$ kPa (3 sig figs)
4. Moles of hydrogen = 0.115 ÷ 2 = 0.0575
   $100\,000 \times 0.0035 = 0.0575 \times 8.31 \times T$ so $T = 732$ K (3 sig figs)
5. $105\,000 \times 0.0016 = n \times 8.31 \times 298$ so $n = 0.0678$
   Mass = moles × 32 = 2.17 g (3 sig figs)
6. $100\,000 \times 0.000122 = n \times 8.31 \times 293$ so $n = 0.00501$
   $M_r = 0.1825 \div 0.00501 = 36.4$ (3 sig figs)
7. $58\,700 \times 0.0001 = n \times 8.31 \times 353$ so $n = 0.00200$
   $M_r = 0.32 \div 0.00200 = 160$ (3 sig figs)
8. (b) Hydrogen fluoride has hydrogen bonding, so deviation will occur because of the assumption IV
   Argon - this is close to conditions for conforming to ideal gas behaviour, so no significant deviation
   Ammonia - this has hydrogen bonding, and this is a high pressure, so deviations due to assumptions I and IV
   Xenon - this is a very low temperature and high pressure, and xenon has a comparatively high M_r, so even though it is monatomic, there will be deviations due to assumptions I and IV.

Acknowledgements: This Factsheet was researched and written by Cath Brown.. ***Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU. ChemistryFactsheets*** *may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber. No part of these Factsheets may be reproduced, stored in a retrieval system, or transmitted, in any other form or by any other means, without the prior permission of the publisher.* ***ISSN 1351-5136***