Chapter 3 Analogue signal processing

3.1 LC resonance filters

Learning objectives:

→ Explain what is meant by the resonant frequency of a parallel LC circuit.

→ Calculate the resonant frequency of a parallel LC circuit.

→ Define the *Q* factor of a parallel LC circuit.

Capacitors and inductors in ac circuits

Every radio or TV receiver uses capacitors and inductors to filter out unwanted signals. A radio or TV aerial detects carrier waves of many different frequencies. If unwanted signals were not filtered out, the wanted signal would be impossible to detect amidst a large number of other signals. Filter circuits contain capacitors and inductors because:

capacitors filter out low-frequency alternating currents. The current through a capacitor for a given alternating pd decreases if the frequency of the pd is decreased. This is because the charge on the capacitor alternates in phase with the pd, so the rate of flow of charge is reduced if the frequency is reduced. (Recall that the capacitance C of a capacitor is the charge it stores per unit pd. The unit of capacitance is the farad, where one farad is equal to one coulomb per volt.)

inductors filter out high-frequency alternating currents. An inductor is an electrical coil, and the current in it produces a magnetic field. A back emf is induced in a coil when there is an alternating current in the coil. The back emf:

- opposes the pd applied to the coil (in accordance with Lenz's law)
- increases if the frequency of the alternating current increases (in accordance with Faraday's law).

So for a given alternating pd across an inductor, the alternating current in the coil decreases if the frequency of the alternating pd is increased. (Note that the inductance L of a coil is the emf induced in it per unit rate of change of current in it. The unit of inductance is the henry (H), where 1 H is equal to $1\,\mathrm{V\,A^{-1}\,s}$ or Ω s.)

Link

Back emf was looked at in Topic 25.2, The laws of electromagnetic induction, of Year 2 of the *AQA Physics* student book.

The parallel LC circuit

Figure 1 shows a receiver aerial connected to one end of a parallel combination of a capacitor and a coil with the other end earthed. The capacitor blocks low-frequency signals and allows high-frequency signals to pass through to earth. The inductor allows low-frequency signals through to earth and blocks high-frequency signals. So the low-frequency signals and the high-frequency signals from the aerial pass through to earth, but a middle band of frequencies do not. The signals that do not pass to earth each create a pd across the parallel combination and can therefore be detected by a suitable receiver connected across the combination.

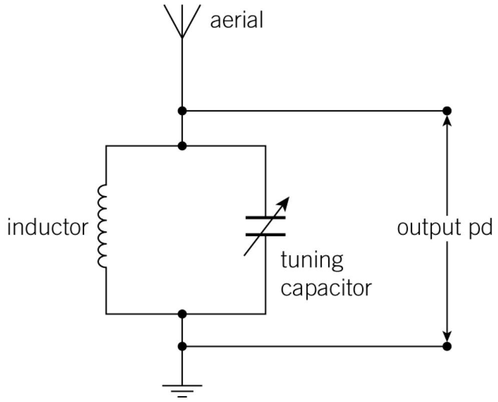

Figure 1 *A parallel LC circuit*

Figure 2 shows how the root-mean-square (rms) pd across the parallel LC circuit varies with frequency. The rms pd has a maximum value at a particular frequency, called the **resonant frequency** of the circuit. At this frequency, the circuit is in resonance because the back emf of the coil and the capacitor pd act together to increase the pd across the combination.

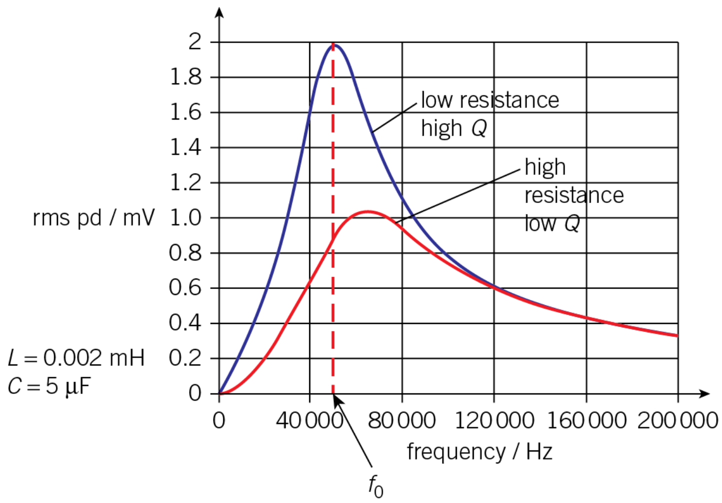

Figure 2 *The frequency response of a parallel LC circuit*

For a parallel LC circuit:

$$\textbf{its resonant frequency } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where L is the inductance of the inductor and C is the capacitance of the capacitor.

When a radio or TV receiver is adjusted to a particular station or channel, the adjustment changes the resonant frequency of the LC filter circuit by changing the inductor (or the inductance of a variable inductor) or the capacitor (or the capacitance of a variable capacitor) in the circuit. The circuit is tuned in when the resonant frequency of the LC circuit is equal to the carrier frequency of the signal from the required station.

Worked example

Calculate the inductance needed for a parallel LC circuit to have a resonant frequency of 100 kHz if $C = 15\,\text{nF}$.

Solution

$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ gives $f_0^{\,2} = \dfrac{1}{(2\pi)^2 LC}$

Rearranging this equation gives

$$L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \times 100 \times 10^3)^2 \times 15 \times 10^{-9}} = 0.17\,\text{mH}$$

Investigating the parallel LC filter

The frequency response of a parallel LC filter can be investigated by connecting a signal generator (in series with a resistor) across the parallel combination. The signal generator is adjusted to change the frequency of the alternating pd and to ensure that the rms value of the alternating pd is kept constant. An oscilloscope connected across the parallel combination can be used to measure the pd across the filter for different frequencies. The graph in Figure 2 shows how the rms value of the alternating pd varies with frequency.

The *Q* factor of a parallel LC circuit

A parallel LC circuit connected to an aerial detects a signal of frequency equal to the resonant frequency more effectively than signals of any other frequency. However, other signals near the resonant frequency are not filtered out completely if the resistance of the LC circuit is too large. This is because the resistance of the circuit affects the width of the frequency response curve in the same way that the frequency response curve of a mass–spring system is affected by drag forces.

Figure 2 shows the effect of resistance on the frequency response graph of a parallel LC circuit. The graph shows that the less resistance there is in the circuit, the sharper and taller is the curve. The sharpness of the curve defines the quality or the ***Q* factor** of the circuit as follows.

The bandwidth f_B of the circuit is defined as the difference between the frequencies (either side of the peak) at $\sqrt{2}$ of the maximum pd. Note that a pd at $\sqrt{2}$ of the maximum pd corresponds to signal power equal to 50% of the maximum power. The points on the response curve at $\sqrt{2}$ of the maximum pd are called the **50% energy points** of the curve.

The Q factor is the ratio of the resonant frequency to the bandwidth:

$$Q \text{ factor} = \frac{f_0}{f_B}$$

- The greater the *Q* factor, the more selective the circuit is in terms of tuning in to a particular signal frequency.

- The Q factor of a parallel LC circuit is increased by using a coil of low resistance and/or large inductance. However, if the inductance is too large, the capacitance needs to be reduced to tune in to a particular frequency, and if the capacitance is too low, it will be affected by stray capacitance effects.

Comparison of a mass–spring system with a parallel LC circuit

James Maxwell worked out the frequency f of the oscillations of an electrical circuit containing a capacitor and a coil using the equation $f = \frac{1}{2\pi\sqrt{LC}}$, where L is the inductance of the coil and C is the capacitance of the capacitor.

The induced emf in the coil $V_L = L\frac{dI}{dt}$ (from the definition of inductance), where I is the current in the circuit. The unit of inductance is the henry (H).

The pd across the capacitor $V_C = \frac{Q}{C}$, where Q is the capacitor charge and C is the capacitance.

An LC circuit is the electrical equivalent of an object on a spring in forced oscillation.

- The inductor is like the mass of the object because inductance and mass both tend to oppose changes to the system.
- The capacitor is like the spring supporting the object because capacitors and springs both tend to restore the system to equilibrium.
- The resistance of an electrical circuit acts like the resistive forces such as drag in a mass–spring system because resistance and drag both cause the energy of the oscillating system to be dissipated.

1 For an object of mass m oscillating on a spring with negligible drag forces, its acceleration at displacement s from equilibrium is given by the simple harmonic motion equation $\frac{d^2s}{dt^2} = -\frac{k}{m}s$, where k is the spring constant. Therefore, for a mass on a spring, the natural frequency of its oscillations $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

When a lightly damped mass–spring system undergoes forced oscillations, resonance occurs if the frequency of the forced oscillations is equal to $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

2 For a parallel LC circuit with negligible resistance, the charge flow in the circuit has a natural frequency of oscillation corresponding to the capacitor pd $V_C\left(=\frac{Q}{C}\right)$ being equal to the inductor pd $V_L\left(=-L\frac{dI}{dt}\right)$.

Therefore, at the resonant frequency f_0, $\frac{Q}{C} = -L\frac{dI}{dt}$.

Rearranging this equation gives $\frac{dI}{dt} = -\frac{Q}{LC}$.

Because the current is equal to the rate of change of charge on the capacitor, then $I = \frac{dQ}{dt}$, so $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$.

Hence, $\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$, which is the simple harmonic motion equation $\frac{d^2Q}{dt^2} = (-2\pi f)^2 Q$, where $(2\pi f)^2 = \frac{1}{LC}$. Rearranging the equation $(2\pi f)^2 = \frac{1}{LC}$ gives the frequency of oscillations of the charge in the circuit, $f = \frac{1}{2\pi\sqrt{LC}}$.

When an alternating pd is applied to the circuit, the charge in the circuit undergoes forced oscillations. Resonance occurs if the frequency of the forced oscillations is equal to $\frac{1}{2\pi\sqrt{LC}}$.

QUESTION: Describe the current in a parallel LC circuit when the circuit operates at its resonant frequency.

Summary questions

1 Calculate the inductance of an inductor in a parallel LC circuit designed to detect radio signals of frequency 200 kHz if the capacitor has a capacitance of 40 nF.

2 The capacitor in Question **1** is a variable capacitor. Describe and explain how the capacitance must be changed to tune in to radio signals of higher frequency.

3 **a** State what is meant by the bandwidth of a parallel LC circuit.

b A parallel LC circuit has a resonant frequency of 92.0 MHz and a bandwidth of 200 kHz. Calculate the frequency of the 50% energy points of the circuit.

4 **a** A parallel LC filter has a resonant frequency of 500 kHz and a Q factor of 5. The filter is in series with a resistor and a signal generator which is adjusted so the rms pd of its output is constant. Sketch a graph to show how the voltage across the filter varies with frequency.

b Describe how the curve would differ if the inductor is replaced with an inductor of the greater inductance and a smaller resistance.

3.2 Ideal operation amplifiers

Learning objectives:

→ Define the open-loop gain of an operational amplifier.

→ State the input and output resistance of an ideal operational amplifier.

→ Explain how a comparator circuit works.

Operational amplifier characteristics

As outlined in Topic 2.2, the output pd from an electronic sensor is supplied to a processing unit which operates an output device. The processing unit may be designed to compare the output pd from the sensor with another pd, or it may need to amplify or invert the output pd from the electronic sensor.

An **amplifier** is a circuit designed to produce an output pd that is proportional to the input pd supplied to it. The **voltage gain** of an amplifier is the ratio of the output pd to the input pd. In effect, the amplifier multiplies the input pd by a constant (i.e., the voltage gain). An amplifier that is used for a specific operation is called an operational amplifier. Different types of **operational amplifier** (op-amp) circuits include:

- comparators that compare two different pds
- inverting amplifiers that invert a pd and multiply it by a constant
- non-inverting amplifiers that multiply a pd by a positive constant.

An operational amplifier is an integrated circuit that includes transistors, resistors, and capacitors formed on a small silicon chip. It has two input terminals and a single output terminal. It also needs a power supply with a positive, a zero, and a negative line (e.g., +15 V, 0 V, and −15 V for a 741 op-amp). The pds applied to the input terminals and the output pd are measured relative to the 0 V (i.e., earthed) line of the power supply. The power supply and its terminals are not normally shown on circuit diagrams.

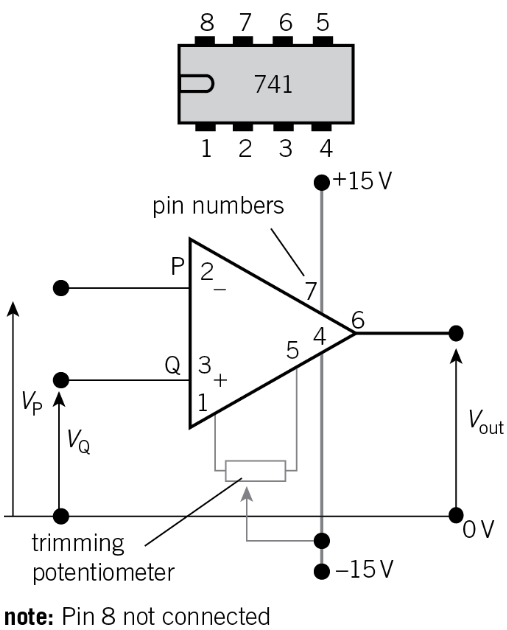

Figure 1 *The 741 op-amp*

The input terminals are called the inverting input, P, and the non-inverting input, Q.
The output pd has:

- the opposite polarity to the pd applied at P if Q is earthed
- the same polarity as the pd applied at Q if P is earthed.

For pd V_- applied to input P, and pd V_+ applied to input Q:

$$\textbf{the output pd } V_{out} = A_0 (V_+ - V_-)$$

where A_0 is called the **open-loop gain** of the op-amp and is its gain with no additional components. The right hand side of equation for the output pd is called the **open loop transfer function.** Figure 2 shows the open-loop characteristics as a graph of V_{out} against $(V_+ - V_-)$.

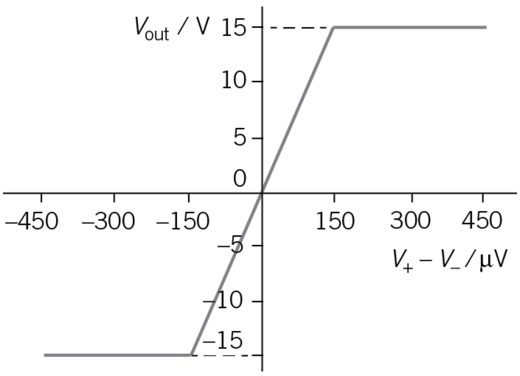

Figure 2 *Open-loop characteristics*

Notes

1 The open-loop gain of an op-amp for direct potential differences is typically of the order of 10^5. This is represented on the graph by the sloped section. An ideal op-amp would have an infinite open-loop gain.

2 The output pd cannot exceed the ±15 V limits determined by the power supply. Therefore, on open-loop, $(V_+ - V_-)$ should not exceed about ±150 μV $\left(= \dfrac{\pm 15\ \text{V}}{10^5} \right)$ if the output pd is not to reach its limits. If it does, the output is said to be saturated.

3 An ideal op-amp has an infinite input resistance and zero output resistance. A 741 op-amp has an input resistance of the order of 1 MΩ. As long as its output does not saturate, the input current is therefore of the order of 10^{-10} A $\left(= \dfrac{150\ \mu\text{V}}{1\ \text{M}\Omega} \right)$.

An ideal op-amp would also have:

- an infinite bandwidth, which means it would amplify all alternating pds by the same factor regardless of the frequency of the alternating pd
- an infinite slew rate, which means that there is no delay between a change to the input pd to the op-amp and the corresponding change to its output pd.

4 As long as the output pd is not saturated, the pd at P differs from the pd at Q by less than 150 μV, so P and Q are virtually at the same pd. If either P or Q are earthed, the terminal that is not earthed is said to be at **virtual earth**, as long as the output pd is not saturated.

The comparator

The open-loop op-amp is a comparator because it can be used directly to compare a pd applied to P with a pd applied to Q. Assuming that the difference in the input pds exceeds 150 μV, the output is either at positive saturation (i.e., the maximum positive output pd, e.g., +15 V) or negative saturation (i.e., the maximum negative output pd, e.g., −15 V). In other words, if:

- the output pd V_{out} is at positive saturation, then $V_+ > V_-$
- the output pd V_{out} is at negative saturation, then $V_- > V_+$.

The comparator may be used to provide a switched response to a gradual change of input pd.

Figure 3 shows a temperature-operated buzzer in which the buzzer sounds if the temperature of the thermistor increases above a pre-set level. This happens because the output pd switches from negative to positive saturation when the pd at P falls below the pd at Q as the thermistor becomes warmer.

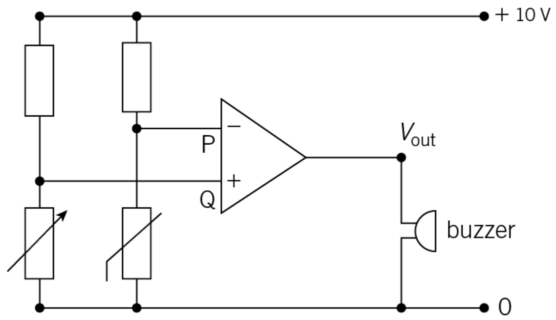

Figure 3 *A temperature-operated buzzer*

The pd at Q can be changed by changing the resistance of the variable resistor. This alters the pre-set level at which the output pd switches.

- If the variable resistor is reduced, the pd at Q is reduced. Therefore, the thermistor needs to reach a higher temperature before the pd at P becomes lower than the pd at Q to make the output pd switch.
- If the variable resistor is increased, the pd at Q is increased. Therefore, the thermistor does not need to reach such a high temperature to make the pd at P lower than the pd at Q. So the output pd switches at a lower temperature.

Therefore, by setting the variable resistor to a suitable resistance, the buzzer switches on when the thermistor temperature increases to the desired temperature.

Summary questions

1 Explain, in relation to an operational amplifier, what is meant by:

- **a** saturation
- **b** virtual earth.

2 The comparator in Figure 4 is used to compare two voltages.

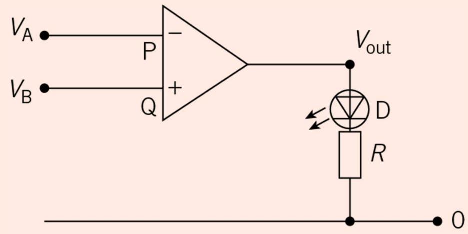

Figure 4

- **a** When the LED lights up, state and explain which voltage V_A or V_B is the greater of the two.
- **b** **i** The output voltage is either +15 V or −15 V depending on which of the two voltages is the greater. When the LED lights up, the pd across the LED terminals is 2.2 V and the current through it is 2.0 mA. Calculate the resistance of resistor *R*.
  - **ii** Explain why a resistor with a lower resistance would not be suitable in this circuit.

3 **a** When the buzzer in Figure 3 is off, state whether the pd at P is greater than or smaller than the pd at Q.

- **b** When the temperature of the thermistor in Figure 3 changes, the buzzer switches on at a certain temperature. State and explain whether this temperature is higher or lower than the initial temperature of the thermistor.

4 Draw a circuit diagram to show how an op-amp can be used to switch a buzzer on when the light intensity on a light-dependent resistor increases to a certain level.

3.3 Operational amplifier circuits

Learning objectives:

→ Describe an inverting and a non-inverting operational amplifier.

→ Calculate the output voltage from a circuit such as an inverting amplifier.

→ Describe what is meant by the bandwidth of an amplifier such as an inverting amplifier.

Negative feedback

Feedback is the process of taking some of the output signal from an amplifier and adding it to the input signal. The effect is to increase or decrease the output signal according to whether the feedback is negative (which reduces the output) or positive which increases the output. Figure 1 shows the basic idea applied to an amplifier.

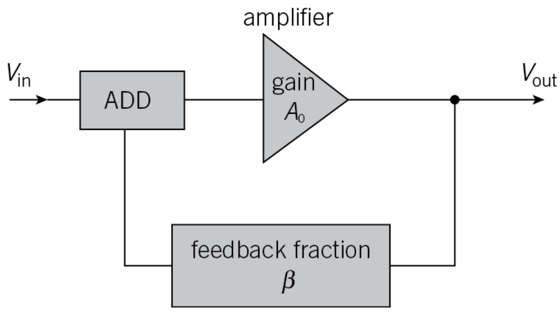

Figure 1 *Feedback*

In Figure 1, an input pd V_{in} is supplied to the amplifier, and the output pd is V_{out}.

Without feedback, $V_{out} = A_o V_{in}$, where A_o is the open-loop gain of the amplifier.

With feedback, a fraction β of the output pd is added to the input pd.

Therefore the total input pd to the amplifier with feedback is $V_{in} + \beta V_{out}$.

Hence the total output pd is $V_{out} = A_o(V_{in} + \beta V_{out})$.

Rearranging this equation gives:

$$\text{the overall voltage gain } \frac{V_{out}}{V_{in}} = \frac{A_o}{1 - \beta A_o}$$

Feedback at a chosen fraction can be supplied using selected resistors without consideration of the open-loop gain of the op-amp. This is because the above equation can be written as $\frac{V_{out}}{V_{in}} = \frac{1}{-\beta}$ as long as the value of βA_o disregarding its sign is much greater than 1 (i.e., $|\beta A_o| >> 1$). Because the value of A_o is typically about 10^5, the open-loop gain of the op-amp does not therefore need to be considered as long as $\beta >> 10^{-5}$.

The condition $|\beta A_o| >> 1$ means that the gain is reduced considerably from its open-loop value by having feedback. However, as explained later in this topic, this has several benefits, which are:

- a wider range for the input pd V_{in} without the output pd saturating at the power supply limits
- an increased frequency range as the frequency range is reduced when the gain is increased
- less distortion of the output waveform and greater stability of the output voltage.

The inverting amplifier

To make an op-amp circuit that can amplify an input pd greater than 150 μV without causing saturation, the voltage gain must be reduced. As explained above, this can be achieved by feeding back a proportion of the output pd to the inverting input of the op-amp. Such feedback is negative feedback because it reduces the voltage gain. Figure 2 shows how negative feedback can be achieved in an op-amp circuit.

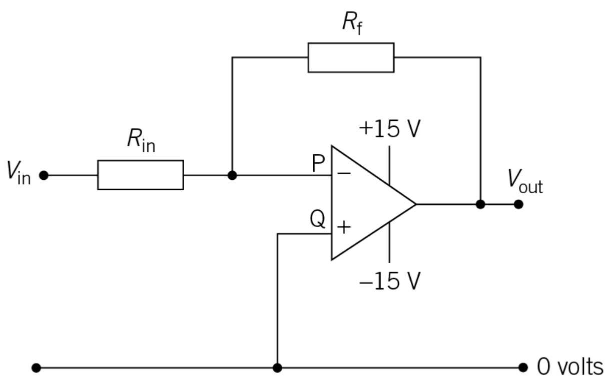

Figure 2 *The inverting amplifier*

- The two resistors form a potential divider between the output terminal of the op-amp and the input terminal.
- The potential divider is used to provide a fixed proportion of the pd between the output and the input to the inverting input of the op-amp. With the non-inverting input earthed, it can be shown that the voltage gain is given by

$$\text{voltage gain } \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

where R_f is the feedback resistor and R_{in} is the input resistor.

To prove that the voltage gain is equal to $-\frac{R_f}{R_{in}}$, consider the inverting amplifier circuit in Figure 2. All pds are measured relative to the zero volt line. For an input pd V_{in}, current I passes between the voltage source at the input and the output of the op-amp through the two resistors. Assuming that the output pd is not saturated, no current enters the op-amp at P, because the input resistance of the op-amp is very high. Also P is virtually at zero potential or 'virtual earth' because Q is at earth potential and the size of the pd between Q and P is less than 150 microvolts. Therefore, because the potential at P, $V_- \approx 0$:

- $V_{in} - V_- = IR_{in}$ gives $V_{in} = IR_{in}$
- $V_- - V_{out} = IR_f$ gives $V_{out} = -IR_f$

Hence the voltage gain is $\dfrac{V_{out}}{V_{in}} = -\dfrac{IR_f}{IR_{in}} = -\dfrac{R_f}{R_{in}}$

Notes

1 The voltage gain of the inverting amplifier depends only on the resistances R_{in} and R_f and is independent of the open-loop voltage gain, A_o.

2 The above equation applies to input pds that are direct voltages and alternating voltages up to a certain frequency as explained at the end of this topic.

3 The inverting amplifier gives an output pd which is inverted compared with the input pd. In other words, when the input pd is positive, the output pd will be negative, and when the input pd is negative, the output pd will be positive.

The summing amplifier

Figure 3 shows an inverting amplifier with three input terminals and three input resistors. Because P is at virtual earth, the current through each input resistor is equal to the corresponding input voltage divided by the input resistance.

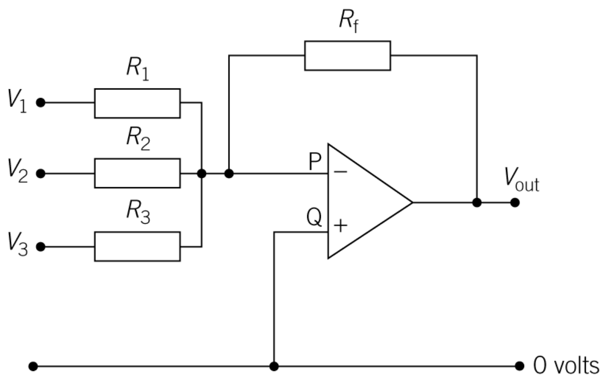

Figure 3 *The summing amplifier*

As long as the output voltage does not saturate, the current I through the feedback resistor is equal to the sum of the currents through the feedback resistors. Hence,

$I = \dfrac{V_1}{R_1} + \dfrac{V_1}{R_2} + \dfrac{V_1}{R_3}$.

Therefore the output pd $V_{out} = -IR_f = -\left(\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right)R_f$.

Choosing input resistances of equal input resistance R therefore gives

$V_{out} = -(V_1 + V_2 + V_3) \times \dfrac{R_f}{R}$.

Therefore

$$\textbf{the output voltage} = \textbf{the sum of the input voltages} \times \frac{\boldsymbol{R_f}}{\boldsymbol{R}}$$

The difference amplifier

The op-amp circuit shown in Figure 4 is a difference amplifier. This is because it amplifies the difference between the two input voltages V_1 and V_2 as long as $R_2 = R_1$ and $R_4 = R_f$. In this case, the output voltage $= (V_2 - V_1) \times \frac{R_f}{R_1}$.

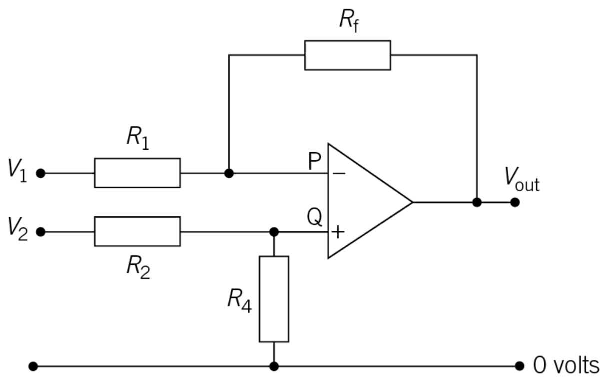

Figure 4 *The difference amplifier*

The non-inverting amplifier

A non-inverting amplifier amplifies an input pd greater than 150 μV without inverting the input pd. Figure 5 shows how this is achieved.

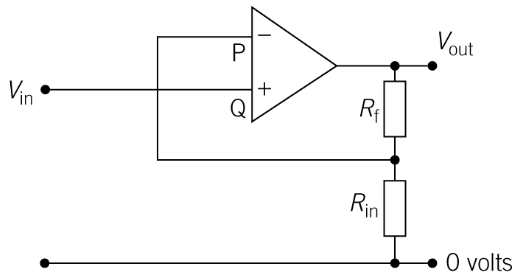

Figure 5 *The non-inverting amplifier*

- The two resistors form a potential divider between the output terminal of the op-amp and 0 V.
- This potential divider is used to provide a fixed proportion of the output pd to the inverting input of the op-amp. With the input voltage V_{in} connected directly to the non-inverting input, it can be shown that the voltage gain is given by

$$\textbf{voltage gain } \frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_{in}}$$

where R_f is the feedback resistor and R_{in} is the input resistor.

To prove the voltage gain $= 1 + \frac{R_f}{R_{in}}$, consider the non-inverter amplifier circuit in Figure 4.

For an input pd V_{in}, current I passes between output of the op-amp and the zero volt line through the two resistors. Assuming the output voltage is not saturated, no current enters the op-amp at P because the input resistance of the op-amp is very high and P is virtually at the same pd as Q (i.e., $V_- \approx V_{in}$). Therefore,

- $V_{out} - V_- = IR_f$ gives $V_{out} - V_{in} = IR_f$
- $V_- - 0 = IR_{in}$ gives $V_{in} = IR_{in}$

Hence $V_{out} - V_{in} = \frac{V_{in} R_f}{R_{in}}$

Rearranging this equation gives voltage gain $\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_{in}}$

The frequency response of an operational amplifier

For an alternating voltage as the input voltage, the voltage gain is independent of frequency up to a particular frequency and decreases at higher frequencies, as shown in Figure 6.

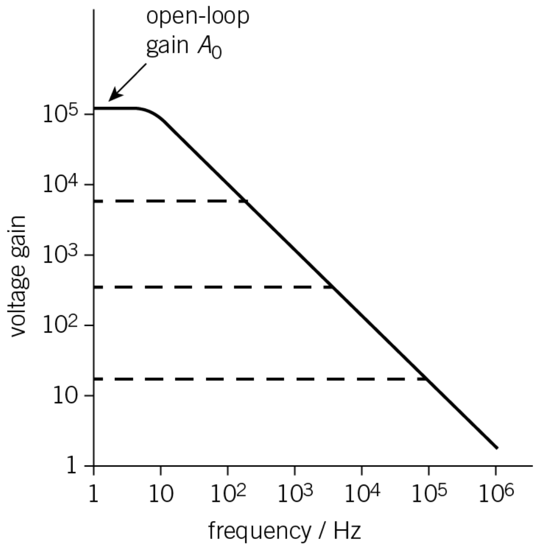

Figure 6 *Frequency response*

The bandwidth is the frequency range over which the voltage gain is constant. Figure 6 shows that for a given device:

the bandwidth × the voltage gain = constant

For example, the bandwidth at a voltage gain of 10 is approximately 10^5 Hz, whereas at a voltage gain of 10^4, the bandwidth is approximately 100 Hz. Therefore, for this device, the bandwidth × the voltage gain = 10^6 Hz.

Summary questions

1 a Draw the circuit diagram for an inverting amplifier.

b If the feedback resistance of an inverting amplifier is 1.0 MΩ, calculate the input resistance that will give a voltage gain of −4.

2 A sinusoidal alternating voltage with a peak voltage of 2.0 V, as shown in Figure 7, is applied to the input of a non-inverting operational amplifier which has a voltage gain of ×5.

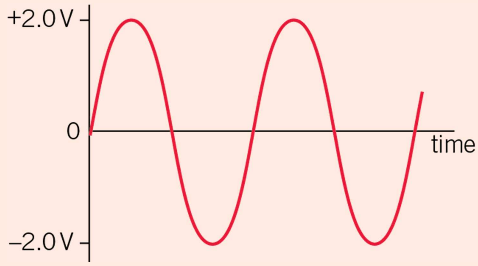

Figure 7

a Copy the axes and draw waveforms to show how the input pd and the output pd vary with time.

b If the output pd saturates at ±15 V, draw a waveform on the same axes to show how the output pd varies with time if the peak voltage of the input pd is increased to 4.0 V.

3 a Explain what is meant by negative feedback in an amplifier.

b Describe and explain how the output pd in Question **2b** would change if the feedback resistor was removed.

4 A summing amplifier has a 200 kΩ feedback resistor and four input terminals A to D with input resistances of 100 kΩ, 200 kΩ, 400 kΩ, and 800 kΩ respectively.

a A voltage of +4.0 V is applied to inputs A and D with the other inputs at zero. Calculate the output voltage.

b The circuit is used to convert a 4-bit byte into an analogue voltage where a ‘1’ at each input has a pd of +4.0 V and a ‘0’ is at zero pd. Calculate:

i the range of the output voltage

ii the least possible change of the output voltage.