Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) (i) $r = \frac{2GM}{c^2} = -\frac{2 \times 6.7 \times 10^{-11} \times 6.0 \times 10^{24}}{(3.0 \times 10^8)^2}$	1	
$= 8.93 \times 10^{-3}$ m $= 8.9$ mm	1	
(ii) Volume of sphere of radius 9 mm $= \frac{4\pi r^3}{3} = \frac{4\pi(9.0 \times 10^{-3})^3}{3}$ $= 3.05 \times 10^{-6}$ m^3	1	
Density $\rho = \frac{mass}{volume} = \frac{6.0 \times 10^{24}}{3.05 \times 10^{-6}}$	1	
$= 2.0 \times 10^{30}$ kg m^{-3}	1	
(b) (i) Michell's idea was put forward as a hypothesis because it was an untested idea, *or* a scientific hypothesis is a suggestion, prediction or untested idea.	1	
Einstein used mathematics to predict his General Theory of Relativity, *or* Schwarzschild predicted (using General Relativity) that light could not escape from a sufficiently massive object.	1	
(ii) Stars photographed near the Sun during a total solar eclipse were displaced relative to each other.	1	
The light from each star was bent by the Sun's gravitational field as it skimmed the Sun.	1	
This observation confirmed the prediction by Einstein.	1	

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(c) FOR *Either* • Scientific knowledge is expanded in scientific projects ✓, for example space exploration has vastly increased our knowledge of the Solar System ✓ *or* • New scientific projects can lead to important new discoveries ✓, for example HST led to the discovery of many stars and galaxies ✓ *or* • New scientific projects can lead to new technologies ✓, for example more powerful space rockets used to reach the Moon made geostationary satellites possible ✓ AGAINST *Either* • Improved living conditions in poorer countries would benefit many more people than a space project would✓, for example provision of clean water ✓ *or* • Cost of the project would be met by taxpayers ✓ and there would be no direct immediate benefits ✓ *or* • Scientific expertise could be better used ✓, for example on developing renewable energy sources to combat global warming ✓	**5**	There are 5 marks available so 5 points to be made. There are two marks for each of the two views, one for identifying a relevant argument and the other for outlining it adequately. Regardless of which of the two views you support, the fifth mark is for providing a valid argument against the view you do not support.

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<u>If FOR</u> *Either* • Improved living conditions in poorer countries would benefit many more people than a space project would – satellite communication has brought benefits to everyone, for example immediate disaster relief *or* • *Cost of the project would be met by taxpayers* – more people would be employed as scientific firms would be contracted to supply parts *or* • *Scientific expertise could be better used* – the demand for scientific expertise could be met by training more scientists <u>If AGAINST:</u> *Either* • *Scientific knowledge is expanded in scientific projects* – there may be cheaper and less risky ways of pursuing knowledge of deep space (for example more advanced and bigger terrestrial telescopes linked to more powerful computers) *or* • New scientific projects can lead to important new discoveries – as above, *or* • New scientific projects can lead to new technologies – as above		Explanation of why you would/would not welcome such a project.
2 **(a)** **(i)** The gravitational field strength at a point is the force per unit mass acting on a small test mass placed at that point in the field.	**1**	The mark would be awarded for $g = \frac{F}{m}$ provided F and m were defined. 'Force on a 1 kg mass' would also be acceptable.
(ii) N kg^{-1} (**not** m s^{-2})	**1**	g has two distinct meanings, and has the same numerical value in both meanings. Treated as an **acceleration**, the unit is m s^{-2}. Treated as a **field strength**, the unit is N kg^{-1}.
(b) **(i)** Since $g_P = g_Q$, it follows that $\frac{GM}{R^2} = \frac{GM_Q}{R_Q^{\,2}}$ $\therefore M_Q = \frac{MR_Q^{\,2}}{R^2} = \frac{M(3R)^2}{R^2}$ since $R_Q = 3R$ giving mass of Q = 9 M	**1** **1** **1**	This solution relies on the relationship between g and the other quantities involved in a radial field: $g = \frac{GM}{R^2}$. You are told in the question that, at the surface of the planets, $g_P = g_Q$.

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(ii) *Line drawn on graph*: • Starts at $3R$, with same initial value of g as existing curve; • is a curve of decreasing negative gradient; • shows the correct $\frac{1}{r^2}$ relationship, checked from points (for example at $6R$, value should be exactly 1.0 vertical square; at $12R$ it should be 0.25 of a vertical square).	**3**	Planet Q has 9 times the mass of planet P. At any given radius, g_Q will be 9 times greater than g_P, since $g \propto M$. Your curve must start at $3R$, since this is the surface radius of Q, and show a $\frac{1}{r^2}$ relationship correctly for full marks.
3 **(a)** *Relevant points about the geosynchronous satellite include*: • It orbits over the Equator. • It maintains a fixed position in relation to the surface of the Earth. • It has a period of 24 hours (the same as the Earth's period of rotation on its axis). *Relevant comparisons with the satellite in a low polar orbit include:* • The geosynchronous satellite enables uninterrupted communication between a transmitter and a receiver whereas the other satellite does not. • Unlike the other satellite, the geosynchronous satellite does not require the use of a steerable dish.	**any 3**	Full marks ought to be obtained easily in part **(a)** if a few facts have been committed to memory. In your answer, take care over the use of language: geosynchronous satellites are **not stationary**, even though they appear to be at rest when viewed from the rotating Earth! They are important as a major means of communicating data. Satellite television relies on their use, so TV aerial dishes can be in fixed positions. A satellite in low polar orbit has a fairly short time period, scanning the Earth several times during the day.
(b) (i) Using the given symbols, the centripetal force equation is $\frac{GMm}{(R+h)^2} = m\,\omega^2\,(R+h)$	**1**	Your answer must be in terms of ω to satisfy the question. The orbit radius is $(R + h)$, because the satellite is at height h above the surface of a planet radius R.
(ii) Substitution of $\omega = \frac{2\pi}{T}$ leads to $\frac{GM}{(R+h)^3} = \frac{4\pi^2}{T^2}$	**1**	The physical basis of this answer is the same as that in Question 2**(b)**, but here the theory is applied to a satellite rather than a planet. The final result is again along the lines of Kepler's 3rd law, $T^2 \propto R^3$, which applies to satellites orbiting planets as well as to planets orbiting the Sun.
and rearrangement of this gives $T^2 = \frac{4\pi^2(R+h)^3}{GM}$	**1**	
(iii) The limiting case is when the satellite orbits the Earth at zero height; $h = 0$	**1**	A lower orbit produces a shorter period. The lowest conceivable orbit is a satellite grazing the surface of the Earth. The period of this would be around 85 min, so any real satellite must have a longer period. Values for R and M are taken from the Data Booklet.
$T^2 = \frac{4\pi^2(R+h)^2}{GM}$ $= \frac{4\pi^2 \times (6.37 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}$	**1**	
gives $T = 5060$ s (= 84.3 min)	**1**	

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(c) The speed of the satellite increases. It loses potential energy but gains kinetic energy.	**1** **1**	A lower satellite travels at a faster speed and has a shorter period. *Alternatively* $v^2 \propto \frac{1}{r}$, from $\frac{GMm}{r^2} = \frac{mv^2}{r}$
4 (a) (i) Gravitational field strength at the surface of the Earth $g_S = \frac{GM}{R^2}$ Gravitational field strength at height h $g = \frac{GM}{(R+h)^2}$ $\therefore \frac{g}{g_s} = \frac{R^2}{(R+h)^2}$, giving $g = g_s\left(\frac{R}{R+h}\right)^2$	**1** **1** **1**	The equation relating the field strength g to R follows from considering the gravitational force acting on a mass m. This is given by either mg or by $\frac{GMm}{R^2}$. Equating the two values produces the result. The Earth may be considered as a point mass M placed at its centre, which is a distance $(R + h)$ from the point at height h above the surface.
(ii) $g = 9.81 \times \left(\frac{6.37 \times 10^6}{(6.37 \times 10^6) + (2.00 \times 10^5)}\right)^2$ $= 9.22\ \text{N kg}^{-1}$	**1** **1**	Direct substitution into the equation $g = \frac{GM}{(R+h)^2}$ from **(a)(i)** produces a swift solution.
(b) *Relevant points include*: • the force of gravity on the astronaut is still mg, where g is the local value of the field strength within the spacecraft; • this force provides the centripetal force to keep the astronaut in orbit; • the astronaut is in free fall, as is the spacecraft; • the astronaut appears weightless because he or she is not supported.	**any 3**	If 'weight' means the pull of gravity acting on an object, then the astronaut is certainly not weightless. The situation is comparable to a lift where all the supporting cables have broken. Both the lift and its contents would be in free fall. Because the contents are unsupported they **appear** weightless, but they are actually falling to Earth at the acceleration of free fall!
5 (a) (i) Area indicated between the curve and the distance axis, which • starts at distance = 1700 km • ends at distance = 5700 km	**2**	Just as the area under a force–distance graph represents work done, the area under a graph of g against distance represents the work done on a 1 kg mass.

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(ii) By counting squares, the area in **(a)(i)** is 90 ± 10 squares Area of one square $= 200\text{ km} \times 0.1\text{N kg}^{-1}$ $= 1.8 \times 10^4\text{ J kg}^{-1}$ Work done on 1 kg $= 90 \times 2.0 \times 10^4$ $= (1.8 \pm 0.2) \times 10^6$ J $\therefore$ change of gravitational potential energy of satellite $= 450 \times 1.8 \times 10^6$ $= (8.1 \pm 1.0) \times 10^8$ J	1 1 1 1	The work done on a 1 kg mass is the change in gravitational potential, ΔV. This is represented by the area you were expected to mark in part **(a)(i)**. The equation $\Delta W = m\,\Delta V$ indicates that the work done on a 450 kg mass will be 450 times greater. *Alternatively*: From the graph, when $r = 1700$ km, $g = 1.77\text{ N kg}^{-1}$. Substitution in $g = \frac{GM}{r^2}$ gives $GM = 5.12 \times 10^{12}$, allowing you to find ΔV by evaluating $V = -\frac{GM}{r}$ for $r = 1700$ km and 4000 km. Application of $\Delta W = m\,\Delta V$ then leads to the same result.
(b) *Relevant points include*: • The satellite must be raised to a point beyond that at which the resultant gravitational field strength of the Earth/Moon is zero • This is much closer to the Moon than to the Earth • The Moon's mass is much less than the Earth's mass • At their surfaces, g_{Moon} is only $\frac{1}{6}$ of g_{Earth} ($1.7\text{ N kg}^{-1} \ll 9.8\text{ N kg}^{-1}$) • The escape speed from the Moon is much less than that from Earth • On returning, the gravitational potential energy needed is much less so the kinetic energy at launch can be much less • The total rocket weight at launch is much greater on Earth	5	Once the returning Moon probe passes the 'neutral point', it will fall to Earth under the influence of the Earth's gravitational pull. The fuel carried must be sufficient to get it beyond this point. The gravitational pull of the Moon is much less than that of the Earth. Both the force and the distance involved are much smaller, so much less work has to be done to escape from the Moon than from Earth. This means that much less fuel will be required for the returning journey. Multi–stage rockets are used to escape from the Earth, so that some weight can be jettisoned as the fuel is used up.
6 (a) Gravitational potential at a point is the work done per unit mass (by an external agent) … to move a small test mass from infinity to the point.	1 1	If a test mass is released from a point an infinite distance from another body, work is done on the test mass **by the gravitational field** of the body as the mass comes closer. Therefore the external agent does not have to provide positive work, indeed the gravitational potential is **always negative**. (Its value is taken to be zero at infinity.)

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(b) At surface of Earth $V_E = -\frac{GM_E}{R_E}$ At surface of Moon $V_M = -\frac{GM_M}{R_M}$ $\therefore V_M = -G \times \frac{M_E}{81} \times \frac{3.7}{R_E} = -\frac{3.7}{81} \times \frac{GM_E}{R_E}$ $= \frac{3.7}{81} \times V_E$ gives gravitational potential at surface of Moon $V_M = 4.57 \times 10^{-2} \times (-63)$ $= -2.88$ MJ kg^{-1} or -2.9 MJ kg^{-1}	1 1 1	In order to satisfy the requirements of the question, you are only allowed to use the data given in the question itself. The first mark is for recognising how to apply the general equation for gravitational potential, $V = -\frac{GM}{R}$, to the Earth and Moon. The second mark is for appreciating how to rearrange the algebra in the two equations, and the third is for evaluating the gravitational potential. Note that $M_M = \frac{M_E}{81}$ and that $R_M = \frac{R_E}{3.7}$.
(c) *Line drawn on graph*: • Starts at (surface of Earth, $V = -63$) and ends at $V = -2.9$, and begins as a curve of decreasing positive gradient … • rises to a value close to (but below) zero … • then falls to surface of Moon … • from a point which is much closer to the Moon than to Earth.	**any 3**	Gravitational potential is a scalar quantity. The total potential at any point along a line joining the Earth and Moon is the sum of the potentials produced by the Earth and Moon separately. The turning point (maximum) of the graph is the 'neutral point', where the resultant gravitational field strength is zero. See Question 5**(b)**, which is about what happens when a Moon probe is brought back to Earth over this 'potential hill'.
7 (a) The force between two point masses • is proportional to the product of the masses • and inversely proportional to the square of their separation.	2	This is Newton's law of gravitation.
(b) (i) Since $V = -\frac{GM}{R}$, it follows that $V \propto \frac{1}{R}$, $\therefore V \times R$ should be constant Calculation of $V \times R$ for the three lines in the table, e.g. $7.0 \times 10^8 \times 19 \times 10^{10}$; (all three values should be -1.33×10^{20}) $\therefore V \propto \frac{1}{R}$	1 1	The question asks you to **show** inverse proportion. The final part of your answer should end with the conclusion that, since $V \times R$ has been shown to be constant, the potential is inversely proportional to the distance from the centre of the Sun.
(ii) It is clear from the formulae $g = \frac{GM}{R^2}$ and $V = (-)\frac{GM}{R}$, both of which apply in a radial field, that the value of g is given by $g = \frac{V}{R}$ in this case. Alternatively since $V = -\frac{GM_S}{R_S}$, $GM_S = 7.0 \times 10^8 \times 19 \times 10^{10}$ $= 1.33 \times 10^{20}$	1	It is necessary to show how the equation $g = \frac{V}{R}$ is arrived at from the equations for g and V, because it is not a general formula that can be applied to all fields. (The general formula is $g = -\frac{\Delta V}{\Delta r}$.)

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From the first line of the table, $g = \frac{V}{R} = \frac{19 \times 10^{10}}{7.0 \times 10^{8}}$ Alternatively $G = \frac{GM_S}{R_S^2} = \frac{1.33 \times 10^{20}}{(7 \times 10^{8})^2}$	1	The value is to be determined for g 'near the surface of the Sun'. Only the first line in the table is relevant; the other two lines apply at larger distances from the Sun.
$= 271$ N kg^{-1} or 270 N kg^{-1}	1	
(iii) At the Earth's position, $V_S = -\frac{GM_S}{R_S}$ $= \frac{1.33 \times 10^{20}}{1.5 \times 10^{11}} = -8.87 \times 10^{8}$ J kg^{-1}	1	Start by finding the gravitational potential due to the Sun in the position of the Earth's orbit. The Earth's absolute potential energy is $M_E \times V_S$. If it is removed to infinity (where the gravitational attraction of the Sun would be zero), the Earth's absolute potential energy would be 0.
Potential energy of Earth $= M_E \times V_S$ $= 6.0 \times 10^{24} \times (-8.87 \times 10^{8})$ $= -5.32 \times 10^{33}$ J	1	
Potential energy needed to escape $= 0 - (-5.32 \times 10^{33}) = 5.32 \times 10^{33}$ J or 5.3×10^{33} J	1	
(iv) Orbital speed v of Earth around Sun $= \frac{2\pi R}{T} = \frac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 3600}$ $= 2.99 \times 10^{4}$ m s^{-1}	1	The Earth takes one year to orbit the Sun. The orbiting Earth already has a large amount of kinetic energy. Suppose the Earth stops when it arrives at infinity. Its E_K has then all been converted to E_P as a contribution to the **minimum** total energy needed to escape.
$E_K = \frac{1}{2}mv^2$ $= \frac{1}{2} \times 6.0 \times 10^{24} \times (2.99 \times 10^{4})^2$ $= 2.68 \times 10^{33}$ J	1	
Energy needed $= (5.32 - 2.68) \times 10^{33}$ $= 2.64 \times 10^{33}$ J or 2.6×10^{33} J	1	
8 (a) (i) Gravitational potential energy of rocket $= -\frac{GMm}{R}$	1	This is (gravitational potential at the Earth's surface) × (mass of rocket).
(ii) In escaping from the Earth, potential energy gained = kinetic energy lost $\therefore \frac{GMm}{R} = \frac{1}{2}mv^2$	1	When the rocket escapes, it is moved an infinite distance from Earth so that the there is no longer any gravitational attraction. At infinity the gravitational potential energy of the rocket is zero ($V = 0$ by definition at infinity) and we presume that the rocket stops moving when it arrives there.
Mass m of rocket cancels, giving escape speed $v = \sqrt{\frac{2GM}{R}}$	1	
(b) From the above equation $v^2 \propto \frac{M}{R}$ $\therefore \left(\frac{v_P}{v_E}\right)^2 = \frac{M_P}{R_P} \times \frac{R_E}{M_E} = \frac{4M_E}{2R_E} \times \frac{R_E}{M_E} = 2$	1	*Alternatively*: escape speed from planet $v = \sqrt{\frac{2GM}{R}}$ $= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 4 \times 5.98 \times 10^{24}}{2 \times 6.37 \times 10^{6}}}$ $= 1.56 \times 10^{4}$ m s^{-1} (15.6 km s^{-1}). (But this method does not make use of the value of 11.2 km s^{-1} given in the question.)
$\therefore$ escape speed from planet $v_P = \sqrt{2}\, v_E = \sqrt{2} \times 11.2$ $= 15.8$ km s^{-1}	1	

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(c) *Relevant points*: • Work is done against air resistance whilst in the atmosphere • Some of the rocket's kinetic energy is converted into thermal energy in heating the rocket and the atmosphere	**2**	Since some of the rocket's kinetic energy is not effective in raising its gravitational potential energy, it will need more kinetic energy at the start. This means that the escape speed would need to be greater than that calculated from the equation in **(a)(ii)**. **Note**, **however**, that it is somewhat misleading to apply this theory to a space rocket. Rockets are not 'thrown' into space as a stone might be; they are accelerated as they rise.