Essential Skills Help

A and B. Laws of indices (powers)

To multiply powers of the same number or variable add the powers

$a^5 \times a^2 = a^7$

$3y^2 \times 4y^3 = 12y^5$

To divide powers of the same number or variable subtract the powers

$a^5 \div a^2 = a^3$

Anything to the power of 1 just equals itself

$3^1 = 3$
$x^1 = x$

Anything to the power of zero is 1 (apart from 0^0, which is undefined)

$3^0 = 1$
$(-2)^0 = 1$
$x^0 = 1, \; x \neq 0$

B. Simplifying expressions with indices

Deal with numbers and letters separately

Remember: $a = a^1$

$2a^2b \times 3ab^3 = 6a^3b^4$

$$\frac{8x^4y^3}{4x^3y^5} = \frac{2x}{y^2}$$

since $8 \div 4 = 2$, $x^4 \div x^3 = x$ and $y^3 \div y^5 = 1/y^2$ i.e. y^2 on the bottom

A. Evaluating Indices

A negative power gives 1 over what it would be if it was a positive power

$2^{-2} = \frac{1}{4}$

$5^{-3} = \frac{1}{125}$

Power $\frac{1}{2}$ means square root

$x^{\frac{1}{2}} = \sqrt{x}$
$4^{\frac{1}{2}} = \sqrt{4} = 2$

Power $\frac{1}{3}$ means cube root

$8^{\frac{1}{3}} = 2$

When you raise a power to a power, multiply the powers

$(x^2)^3 = x^6$

Take care

$(2x^2)^3 = 2^3(x^2)^3 = 8x^6$ (not $\mathbf{2}x^6$)

C. Substitution

Work out what is inside a bracket first

If $x = 3$ and $y = 5$,

$(2x + 3y)^2 = (6 + 15)^2 = 21^2 = 441$

And $\frac{1}{2}y^2 = \frac{1}{2}$ of 5^2 = $\frac{1}{2}$ of 25 = 12.5

Whereas $(\frac{1}{2}y)^2 = 2.5^2 = 6.25$

D. Multiplying out simple brackets

Remember: $- \times - \rightarrow +$

$3(2x - y) - 2(x - 3y)$
$= 6x - 3y - 2x + 6y$
$= 4x + 3y$

$2x(x - 4) = 2x^2 - 8x$

$a(2a - b) + 3b(b - 2a) = 2a^2 - ab + 3b^2 - 6ab$
$= 2a^2 - 7ab + 3b^2$

E. More complex brackets

Expand $(3x - 4)(2x - 3)$

$(3x - 4)(2x - 3) = 6x^2 - 8x - 9x + 12$

$(3x - 4)(2x - 3) = 6x^2 - 17x + 12$

Take care: $-3 \times -4 = +12$

F. Simple factorisation

Make sure you take out **all** common factors

$2x^2 - 4xy = 2x(x - 2y)$

$6a^2b + 9ab^2 = 3ab(2a + 3b)$

G. Factorising quadratics

This method works in all cases

Example: $6x^2 + 25x - 9$

Compare with $ax^2 + bx + c$

$a = 6$, $b = 25$ and $c = -9$

$ac = -54$, so find two factors of -54 which add to $b = 25$

These are 27 and -2

This is the important step. It doesn't matter which order you put these in

Rewrite as $6x^2 + 27x - 2x - 9$

Factorise in pairs

$= 3x(2x + 9) - 1(2x + 9)$

$= (3x - 1)(2x + 9)$

If you write $6x^2 - 2x + 27x - 9$

you get $2x(3x - 1) + 9(3x - 1)$

$= (2x + 9)(3x - 1)$

which is the same answer.

Useful result:

Difference of Two Squares
$a^2 - b^2 = (a - b)(a + b)$

$x^2 - 9 = (x - 3)(x + 3)$

$9x^2 - 16 = (3x)^2 - (4)^2$
$= (3x - 4)(3x + 4)$

H. Solving linear equations

1. Solve 4(x + 3) – 2(x – 5) = 46

Multiply out the brackets

Take care

$$4x + 12 - 2x + 10 = 46$$

Collect terms

$$2x + 22 = 46$$

Rebalance to make an "x side" and a "number side"

$$2x = 46 - 22$$

Tidy $2x = 24$

Divide by 2 $x = 12$

2. Solve $5x - 4 = 3x + 12$

Rebalance to make an "x side" and a "number side"

$$5x - 3x = 12 + 4$$

Tidy $2x = 16$

Divide by 2 $x = 8$

3. Solve $\frac{x+3}{2} = \frac{x-1}{3}$

To get rid of both denominators (bottoms) multiply by 6

$$3(x + 3) = 2(x - 1)$$

(Some of you will call this cross multiplication)

Expand the brackets

$$3x + 9 = 2x - 2$$

"Move terms" $3x - 2x = -2 - 9$

Tidy $x = -11$

J. Solving quadratic equations: Factorisation

1. Solve $x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

$x = 3$ or -4

2. Solve $3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or 2

I. Simultaneous equations

$$\left.\begin{aligned} 3x + 2y &= 7 \\ 4x + y &= 6 \end{aligned}\right\}$$

Method 1, By Elimination

Multiply bottom equation by 2 to get equal amounts of y top and bottom

$$\left.\begin{aligned} 3x + 2y &= 7 \\ 8x + 2y &= 12 \end{aligned}\right\}$$

Take top equation from bottom equation to eliminate y:-

$5x = 5$ so $x = 1$

Put x = 1 into one of the equations to find y.

$y = 2$

Answer: $x = 1, y = 2$

Method 1, By Substitution

It is easy to get y in terms of x from the second equation

$$y = 6 - 4x$$

Now substitute this into the first equation

$$3x + 2(6 - 4x) = 7$$
$$3x + 12 - 8x = 7$$
$$-5x + 12 = 7$$
$$5 = 5x$$
$$x = 1$$

and proceed as before

This method is only really worth using if you can get one of the variables in terms of the other easily.

K. Solving quadratic equations: The formula

If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

L. Rearranging formulae

Do this exactly the same way as solving equations. The only difference is that the letters don't collect together so you are left with an **expression** as your answer, rather than a number.

1. Make x the subject of the formula
$y = mx + c$

Take c from both sides

$$y - c = mx$$

Now divide **both sides** by m

$$x = \frac{y - c}{m}$$

Notice: all of y – c was divided by m

2. Make r the subject of the formula
$V = \frac{1}{3}\pi r^2 h$

First get rid of the fraction by multiplying both sides by 3

$$3V = \pi r^2 h$$

π and h are both multiplying the r^2 so we need to divide both sides by πh.

$$r^2 = \frac{3V}{\pi h}$$

Now square root both sides to get

$$r = \sqrt{\frac{3V}{\pi h}}$$

Useful hints:

$y = \frac{1}{2}x + 5$ becomes $2y = x + \mathbf{10}$ when you multiply by 2. **Every term** becomes twice as big.

Keep all letters in the same case as in the question i.e. keep capitals as capitals. In 2 above the V is a capital (not v). This avoids confusion if a formula contains both e.g. A and a.

M. Surds

To simplify surds like √50, **look for square number factors.**

$$\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$
$$\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$$
$$\sqrt{50} + \sqrt{98} = 5\sqrt{2} + 7\sqrt{2} = 12\sqrt{2}$$

If a surd is on the bottom then multiply top and bottom by it

$$\frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5}$$

You can't add surds like this: $\sqrt{2} + \sqrt{3}$

Only add if they're the same:
$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$

Example

$3\sqrt{50} + 2\sqrt{18} = 3\sqrt{25}\sqrt{2} + 2\sqrt{9}\sqrt{2}$

$= 3 \times 5\sqrt{2} + 2 \times 3\sqrt{2}$

$= 15\sqrt{2} + 6\sqrt{2}$

$= 21\sqrt{2}$

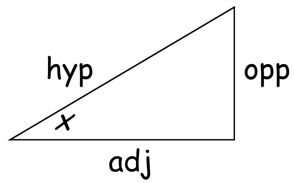
N & O. Trigonometry

For a right angled triangle:

$$\sin x = \frac{opp}{hyp}$$

$$\cos x = \frac{adj}{hyp}$$

$$\tan x = \frac{opp}{adj}$$



S O H C A H T O A