

GCSE Mathematics

Practice Tests: Set 5

Paper 2H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Change 4.5 km^2 to m^2 .

$$1 \text{ km} = 1000 \text{ m}$$
$$\therefore 1 \text{ km}^2 = 1000000 \text{ m}^2$$

$$\dots\dots\dots 4500000 \dots\dots\dots \text{m}^2$$

(2)

(Total 4 marks)

2. Carol and Delia share some money in the ratio 3 : 8
Delia gets £325 more than Carol gets.

Work out how much Carol gets.

$$8 - 3 = 5$$
$$325 \div 5 = 65$$
$$3 \times 65 = 195$$

$$\pounds \dots\dots\dots 195 \dots\dots\dots$$

(Total 2 marks)

3. One day Sadie and Gohil both buy petrol and oil from the same petrol station.

Sadie buys 30 litres of petrol and 4 litres of oil.
Sadie pays a total £46.00

Gohil buys 24 litres of petrol and 8 litres of oil.
Gohil pays a total of £45.20

Find the cost of one litre of petrol and the cost of one litre of oil.

$$30P + 4o = 4600 \quad \times 2$$

$$24P + 8o = 4520$$

$$60P + 8o = 9200$$

$$\therefore 36P = 4680$$

$$\therefore P = 130$$

$$30 \times 130 + 4o = 4600$$

$$\therefore 4o = 700$$

$$\therefore o = 175$$

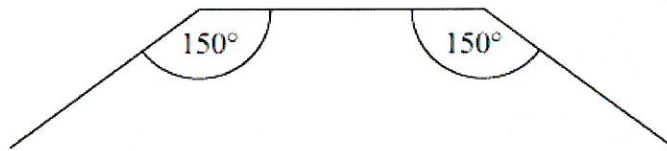
Petrol £ 1.30

Oil £ 1.75

(Total 5 marks)

4. The diagram shows 3 sides of a regular polygon.

Diagram **NOT**
accurately drawn



Each interior angle of the regular polygon is 150° .

Work out the number of sides of the regular polygon.

$$\text{EXTERIOR ANGLE} = 180 - 150 = 30^\circ$$

$$\text{NUMBER OF SIDES} = \frac{360}{30} = 12$$

..... 12

(Total 3 marks)

5. Viv wants to invest £2000 for 2 years in the same bank.

<p>The International Bank</p> <p>Compound Interest</p> <p>4% for the first year 1% for each extra year</p>

<p>The Friendly Bank</p> <p>Compound Interest</p> <p>5% for the first year 0.5% for each extra year</p>
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At the end of 2 years, Viv wants to have as much money as possible.

Which bank should she invest her £2000 in?

$$\text{INTERNATIONAL: } 1.04 \times 1.01 \times 2000 = \text{£}2100.80$$

$$\text{FRIENDLY: } 1.05 \times 1.005 \times 2000 = \text{£}2110.50$$

SHE SHOULD INVEST IN THE FRIENDLY BANK

(Total 4 marks)

6. Malcolm knows he eats too many biscuits.
He wants to eat less than 30 biscuits a week.

On Monday, Malcolm ate some biscuits.

On Tuesday, he ate half as many biscuits as on Monday.

On Wednesday, he ate five biscuits less than on Tuesday.

On Thursday, he ate 9 biscuits.

On Friday, Saturday and Sunday he ate no biscuits.

$$\begin{array}{l} x \\ \frac{1}{2}x \\ \frac{1}{2}x - 5 \\ 9 \end{array}$$

Malcolm ate less than 30 biscuits from Monday to Sunday.

What is the biggest number of biscuits Malcolm could have eaten on Monday?

$$x + \frac{1}{2}x + \frac{1}{2}x - 5 + 9 < 30$$

$$\therefore 2x + 4 < 30$$

$$\therefore 2x < 26$$

$$\therefore x < 13$$

..... 12 biscuits

(Total 4 marks)

7. In a sale the normal price of a book is reduced by 10%.
The sale price of the book is £4.86

Calculate the normal price of the book.

$$\text{SALE PRICE} = 90\% \text{ OF NORMAL PRICE}$$

$$4.86 = 0.9 \times \text{NORMAL PRICE}$$

$$\therefore \text{NORMAL PRICE} = \frac{4.86}{0.9} = 5.4$$

£5.40.....

(Total 3 marks)

8. There are 80 singers in a choir.
The cumulative frequency table gives information about the ages of the 80 singers.

Age (A) in years	Cumulative frequency
$20 < A \leq 30$	2
$20 < A \leq 40$	15
$20 < A \leq 50$	30
$20 < A \leq 60$	52
$20 < A \leq 70$	80

- (a) On the grid opposite, draw a cumulative frequency graph for this information.

(2)

- (b) Use your cumulative frequency graph to find an estimate for

- (i) the median age,

$$\frac{55}{(53 - 57)}$$

- (ii) the interquartile range.

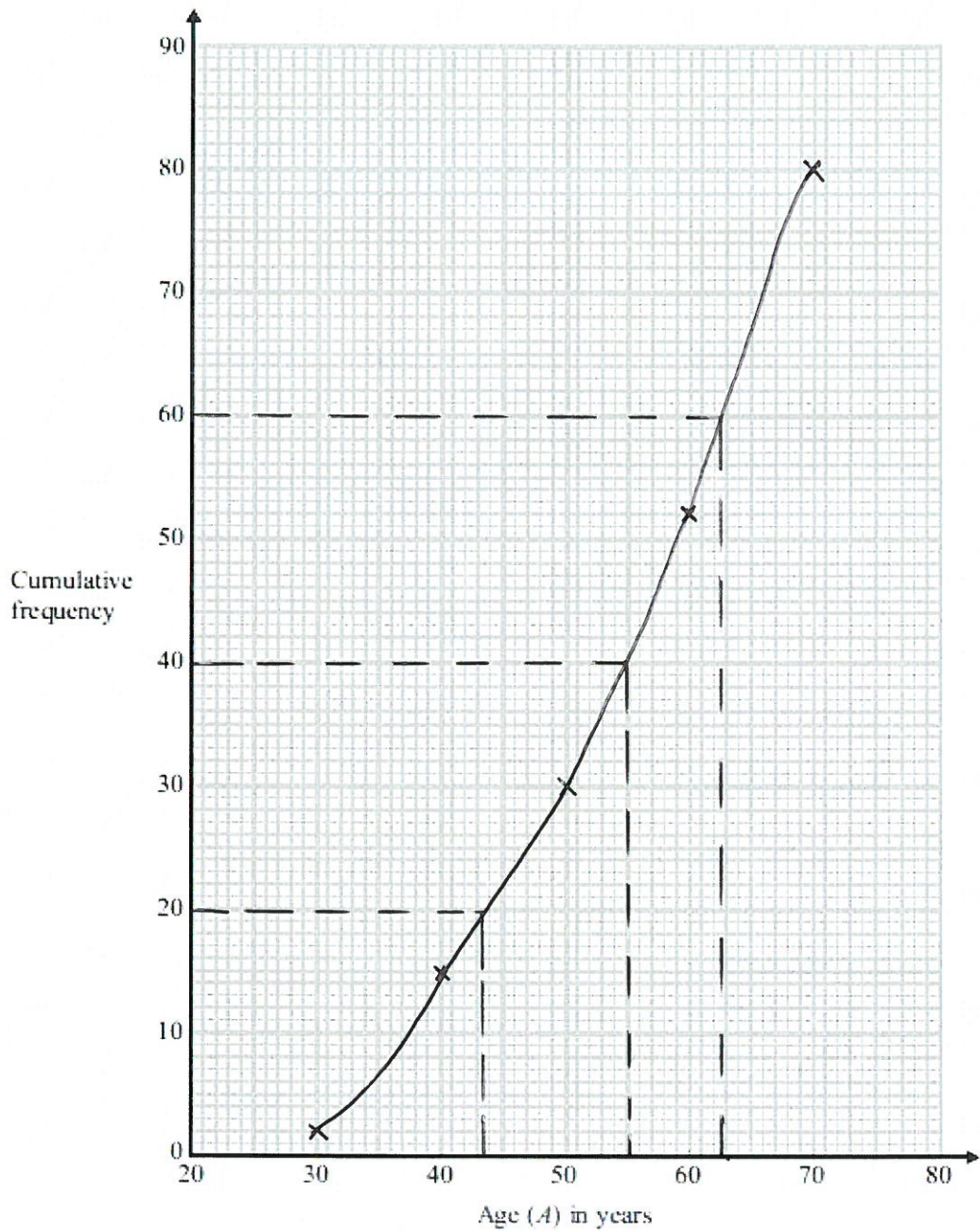
$$62.5 - 43.5$$

$$\frac{19}{(17 - 23)} \quad (3)$$

- (c) Find an estimate for the number of singers over 63 years old.

$$80 - 62.5$$

$$\frac{17.5}{(19 - 23)} \quad (2)$$



(Total 7 marks)

9. A rectangular lawn has a length of $3x$ metres and a width of $2x$ metres. The lawn has a path of width 1 metre on three of its sides.

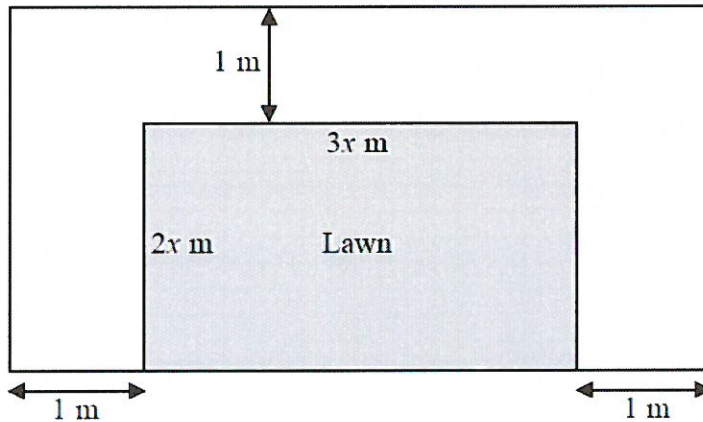


Diagram **NOT** accurately drawn

The total area of the lawn and the path is 100 m^2

- (a) Show that $6x^2 + 7x - 98 = 0$

$$(3x+2)(2x+1) = 100$$

$$\therefore 6x^2 + 3x + 4x + 2 = 100$$

$$\therefore 6x^2 + 7x - 98 = 0$$

(2)

- (b) Calculate the area of the lawn.
Show clear algebraic working.

$$(2x-7)(3x+14) = 0$$

$$\therefore x = \frac{7}{2}, -\frac{14}{3}$$

SINCE $x > 0$, $x = \frac{7}{2}$

$$\text{AREA} = 2x \times 3x = 6x^2 = 6\left(\frac{7}{2}\right)^2$$

$$\dots\dots\dots 73.5 \text{ m}^2$$

(5)

(Total 7 marks)

10. Here is a shape.

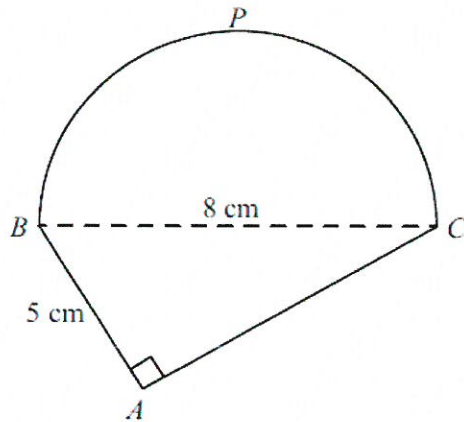


Diagram NOT
accurately drawn

BPC is a semicircle.

ABC is a right-angled triangle.

$BC = 8$ cm.

$AB = 5$ cm.

Work out the perimeter of the shape.

Give your answer correct to 3 significant figures.

$$\text{Arc } BPC = \frac{1}{2} \pi \times 8 = 4\pi$$

$$AC^2 = 8^2 - 5^2 = 39$$

$$\therefore AC = \sqrt{39}$$

$$\therefore \text{PERIMETER} = 4\pi + 5 + \sqrt{39}$$

..... 23.8 cm

(Total 5 marks)

11. Laura is raising money for charity.

She has a game with two sets of cards.

Set A	1	3	5	6	7
Set B	2	4	8	9	

80 students are each going to play Laura's game once.

Each student takes at random one card from each set of cards.
They add the two numbers to get a total score.

Each student pays 70p to play the game.
Laura pays £3 to any student getting a total score of 9.

Show that Laura can expect to make a profit of £20.
You must show all your working.

A \ B	1	3	5	6	7
2	3	5	7	8	9
4	5	7	9	10	11
8	9	11	13	14	15
9	10	12	14	15	16

$$P(9) = \frac{3}{20}$$

$$\text{EXPECTED NUMBER OF WINS} = \frac{3}{20} \times 80 = 12$$

$$\therefore \text{PRIZE MONEY PAID} = 12 \times 3 = \text{£}36$$

$$\text{STUDENTS PAY } 80 \times \text{£}0.70 = \text{£}56$$

$$\therefore \text{PROFIT} = 56 - 36 = \text{£}20$$

(Total 5 marks)

12. (a) $x = 9 \times 10^{2m}$, where m is an integer.

Find, in standard form, an expression for \sqrt{x} .

$$\sqrt{x} = \sqrt{9 \times 10^{2m}} = \sqrt{9} \times \sqrt{10^{2m}} = 3 \times 10^m$$

$$3 \times 10^m$$

(2)

- (b) $y = 9 \times 10^{2n}$, where n is an integer.

Find, in standard form, an expression for $y^{\frac{3}{2}}$.

Give your answer as simply as possible.

$$y^{\frac{3}{2}} = 3 \times 10^n$$

$$\begin{aligned} \therefore y^{\frac{3}{2}} &= (3 \times 10^n)^3 \\ &= 3^3 \times (10^n)^3 \\ &= 27 \times 10^{3n} \\ &= 2.7 \times 10^{3n+1} \end{aligned}$$

$$2.7 \times 10^{3n+1}$$

(3)

(Total 5 marks)

13. The diagram shows a triangular prism with a horizontal rectangular base $ABCD$.

$$AB = 10 \text{ cm. } BC = 7 \text{ cm.}$$

M is the midpoint of AD .

The vertex T is vertically above M .

$$MT = 6 \text{ cm.}$$

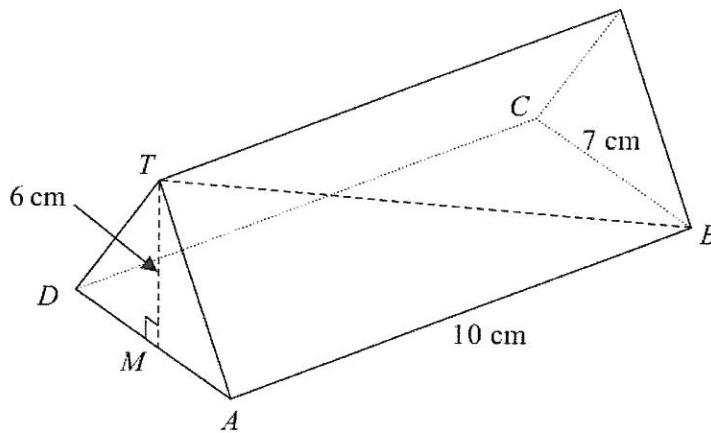


Diagram NOT
accurately drawn

Calculate the size of the angle between TB and the base $ABCD$.

Give your answer correct to 1 decimal place.

$$BM^2 = 10^2 + 3.5^2 = 112.25$$

$$\therefore BM = \sqrt{112.25}$$

$$\tan \alpha = \frac{TM}{BM} = \frac{6}{\sqrt{112.25}}$$

$$\therefore \alpha = \tan^{-1} \left(\frac{6}{\sqrt{112.25}} \right)$$

.....
29.5

(Total 4 marks)

14.

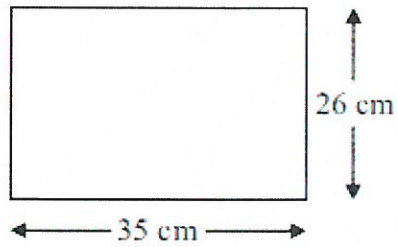


Diagram NOT
accurately drawn

The length of the rectangle is 35 cm correct to the nearest cm.
The width of the rectangle is 26 cm correct to the nearest cm.

$$L_{\text{MAX}} = 35.5$$
$$W_{\text{MAX}} = 26.5$$

Calculate the upper bound for the area of the rectangle.
Write down all the figures on your calculator display.

$$\text{UPPER BOUND OF AREA} = 35.5 \times 26.5$$

$$\dots\dots\dots 940.75 \dots\dots\dots \text{cm}^2$$

(Total 3 marks)

15. f is a function such that

$$f(x) = \frac{1}{x^2 + 1}$$

(a) Find $f\left(\frac{1}{2}\right)$

$$f\left(\frac{1}{2}\right) = \frac{1}{\left(\frac{1}{2}\right)^2 + 1} = \frac{1}{\frac{1}{4} + 1} = \frac{4}{5}$$

$$\frac{4}{5} \text{ or } 0.8$$

(1)

g is a function such that

$$g(x) = \sqrt{x-1} \quad x \geq 1$$

(b) Find $fg(x)$

Give your answer as simply as possible.

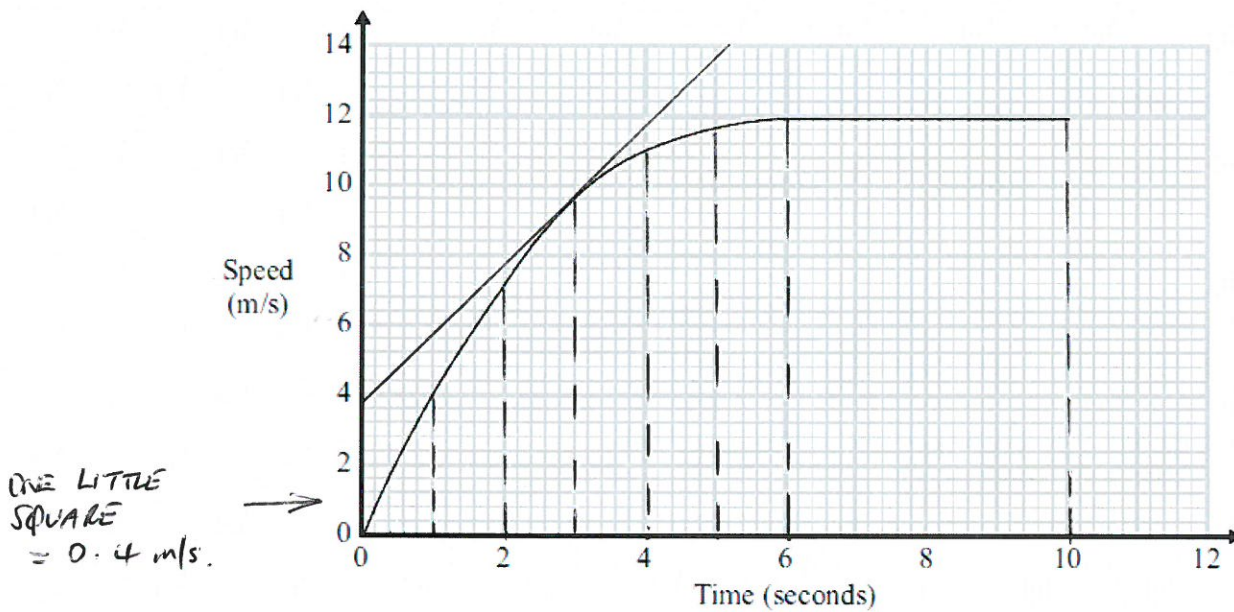
$$fg(x) = \frac{1}{(\sqrt{x-1})^2 + 1}$$
$$= \frac{1}{x-1+1}$$

$$fg(x) = \frac{1}{x}$$

(2)

(Total 3 marks)

16. An athlete is running in a race.
The diagram shows a speed-time graph for the athlete.



- (a) Work out an estimate for the acceleration of this athlete at 3 seconds.

$$\frac{11.6 - 3.8}{4 - 0} = 7.8$$

$$\frac{1.95}{(1.6 - 2.4)} \text{ m/s}^2 \quad (3)$$

The athlete ran the race in 10 seconds.

The area under the graph gives the distance the athlete ran in the 10 seconds.

- (b) Show that this race could have been a 100 metre race.

$$\begin{aligned} \text{AREA} &= \frac{1}{2} \times 1 \times 4 + \frac{1}{2} (4 + 7) + \frac{1}{2} (7 + 9.6) + \frac{1}{2} (9.6 + 11) \\ &\quad + \frac{1}{2} (11 + 11.6) + \frac{1}{2} (11.6 + 12) + 4 \times 12 \end{aligned}$$

$$= 2 + 5.5 + 8.3 + 10.3 + 11.3 + 11.8 + 48$$

$$= 97.2 \quad (96 - 102)$$

EXPECT THIS METHOD TO YIELD AN UNDERESTIMATE

∴ DISTANCE > 97.2m ∴ COULD HAVE BEEN
A 100M RACE.

(3)

(Total 6 marks)

17. Rhodri is investigating the population growth of the rabbits in an enclosure.

He counted the number of rabbits at the start of month 0.
He counted the number of rabbits at the start of month 2.

Here are his results.

Month	Population
0	100
2	200

The population of rabbits is increasing exponentially.

How many rabbits will there be at the start of month 5?
You must show your working.

$$y = ka^x$$

$$x = 0, y = 100 \quad \therefore 100 = k$$

$$y = 100a^x$$

$$x = 2, y = 200 \quad \therefore 200 = 100 \times a^2$$

$$\therefore a^2 = 2$$

$$\therefore a = \sqrt{2}$$

$$\begin{aligned} \text{MONTH 5: } y &= 100 \times (\sqrt{2})^5 = 100 \times 2^{5/2} \\ &= 566 \end{aligned}$$

..... ⁵⁶⁶ rabbits
(or 565) (Total 5 marks)

18.

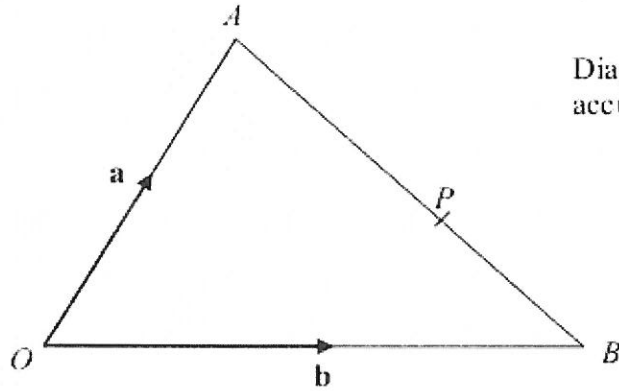


Diagram **NOT** accurately drawn

OAB is a triangle.

$$\vec{OA} = \mathbf{a}$$

$$\vec{OB} = \mathbf{b}$$

(a) Find the vector \vec{AB} in terms of \mathbf{a} and \mathbf{b} .

$$\vec{AB} = \underline{\underline{-\mathbf{a} + \mathbf{b}}} \quad (1)$$

P is the point on AB such that $AP : PB = 3 : 2$

$$\therefore AP = \frac{3}{5} AB$$

(b) Show that $\vec{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

$$\begin{aligned} \vec{OP} &= \vec{OA} + \vec{AP} = \underline{\underline{\mathbf{a}}} + \frac{3}{5} \vec{AB} = \underline{\underline{\mathbf{a}}} + \frac{3}{5} (\underline{\underline{-\mathbf{a}}} + \underline{\underline{\mathbf{b}}}) \\ &= \underline{\underline{\mathbf{a}}} - \frac{3}{5} \underline{\underline{\mathbf{a}}} + \frac{3}{5} \underline{\underline{\mathbf{b}}} \\ &= \frac{2}{5} \underline{\underline{\mathbf{a}}} + \frac{3}{5} \underline{\underline{\mathbf{b}}} \\ &= \frac{1}{5} (2\underline{\underline{\mathbf{a}}} + 3\underline{\underline{\mathbf{b}}}) \quad (3) \end{aligned}$$

(Total 4 marks)

19. Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \text{ is an even number}$$

for all positive integer values of n .

$$(2n+1)^2 = (2n+1)(2n+1) = 4n^2 + 4n + 1$$

$$(2n+1)^2 - (2n+1) = (4n^2 + 4n + 1) - (2n + 1)$$

$$= 4n^2 + 4n + 1 - 2n - 1$$

$$= 4n^2 + 2n$$

$$= 2(2n^2 + n)$$

MULTIPLE OF 2 \therefore EVEN

QED.

(Total 3 marks)

TOTAL FOR PAPER IS 80 MARKS