

**1**    **a**  $3x = 2x + 1$

$$x = 1$$

$$\therefore x = 1, y = 3$$

**b**  $x - 6 = \frac{1}{2}x - 4$

$$x = 4$$

$$\therefore x = 4, y = -2$$

**c**  $2x + 6 = 3 - 4x$

$$x = -\frac{1}{2}$$

$$\therefore x = -\frac{1}{2}, y = 5$$

**d** subtracting

$$y + 4 = 0$$

$$y = -4$$

$$\therefore x = 7, y = -4$$

**e**  $2x + 4y + 22 = 0$

$$2x - 3y + 1 = 0$$

subtracting

$$7y + 21 = 0$$

$$y = -3$$

$$\therefore x = -5, y = -3$$

**f**  $6x + 6y + 8 = 0$

$$15x - 6y - 15 = 0$$

adding

$$21x - 7 = 0$$

$$x = \frac{1}{3}$$

$$\therefore x = \frac{1}{3}, y = -\frac{5}{3}$$

**2**    **a**  $x + 2 = x^2 - 4$

$$x^2 - x - 6 = 0$$

$$(x + 2)(x - 3) = 0$$

$$x = -2 \text{ or } 3$$

$$\therefore (-2, 0) \text{ and } (3, 5)$$

**b**  $4x + 11 = x^2 + 3x - 1$

$$x^2 - x - 12 = 0$$

$$(x + 3)(x - 4) = 0$$

$$x = -3 \text{ or } 4$$

$$\therefore (-3, -1) \text{ and } (4, 27)$$

**c**  $2x - 1 = 2x^2 + 3x - 7$

$$2x^2 + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

$$x = -2 \text{ or } \frac{3}{2}$$

$$\therefore (-2, -5) \text{ and } (\frac{3}{2}, 2)$$

**3**    **a** subtracting

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x = -1 \text{ or } 2$$

$$\therefore x = -1, y = 4$$

$$\text{or } x = 2, y = 7$$

**b** adding

$$2x^2 - 7x + 3 = 0$$

$$(2x - 1)(x - 3) = 0$$

$$x = \frac{1}{2} \text{ or } 3$$

$$\therefore x = \frac{1}{2}, y = -\frac{7}{2}$$

$$\text{or } x = 3, y = -6$$

**c**  $y = 2x - 5$

sub

$$x^2 + (2x - 5)^2 = 25$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \text{ or } 4$$

$$\therefore x = 0, y = -5$$

$$\text{or } x = 4, y = 3$$

**d**  $y = 2x + 10$

sub.

$$x^2 + 2x(2x + 10) + 15 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

$$x = -3 \text{ or } -1$$

$$\therefore x = -3, y = 4$$

$$\text{or } x = -1, y = 8$$

**e**  $y = 1 - x$

sub.

$$x^2 - 2x(1 - x) - (1 - x)^2 = 7$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\therefore x = -2, y = 3$$

$$\text{or } x = 2, y = -1$$

**f**  $y = 1 - x$

sub.

$$3x^2 - x - (1 - x)^2 = 0$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = -1 \text{ or } \frac{1}{2}$$

$$\therefore x = -1, y = 2$$

$$\text{or } x = \frac{1}{2}, y = \frac{1}{2}$$

**g**  $y = 4 - x$

sub.

$$2x^2 + x(4 - x) + (4 - x)^2 = 22$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \text{ or } 3$$

$$\therefore x = -1, y = 5$$

$$\text{or } x = 3, y = 1$$

**h**  $x = 2y$

sub.

$$(2y)^2 - 4y - y^2 = 0$$

$$3y^2 - 4y = 0$$

$$y(3y - 4) = 0$$

$$y = 0 \text{ or } \frac{4}{3}$$

$$\therefore x = 0, y = 0$$

$$\text{or } x = \frac{8}{3}, y = \frac{4}{3}$$

**i**  $y = 3 - \frac{3}{2}x$

sub.

$$x^2 + x(3 - \frac{3}{2}x) = 4$$

$$x^2 - 6x + 8 = 0$$

$$(x - 2)(x - 4) = 0$$

$$x = 2 \text{ or } 4$$

$$\therefore x = 2, y = 0$$

$$\text{or } x = 4, y = -3$$

j  $y = 2x - 3$

sub.

$$2x^2 + (2x-3) - (2x-3)^2 = 8$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

$$x = 2 \text{ or } 5$$

$$\therefore x = 2, y = 1$$

$$\text{or } x = 5, y = 7$$

k  $y = 2x - 7$

sub.

$$x^2 - x(2x-7) + (2x-7)^2 = 13$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$x = 3 \text{ or } 4$$

$$\therefore x = 3, y = -1$$

$$\text{or } x = 4, y = 1$$

l  $y = 5 - 3x$

sub.

$$x^2 - 5x + (5 - 3x)^2 = 0$$

$$2x^2 - 7x + 5 = 0$$

$$(2x-5)(x-1) = 0$$

$$x = 1 \text{ or } \frac{5}{2}$$

$$\therefore x = 1, y = 2$$

$$\text{or } x = \frac{5}{2}, y = -\frac{5}{2}$$

m  $x = 2y + 10$

sub.

$$3(2y+10)^2 - y(2y+10) + y^2 = 36$$

$$y^2 + 10y + 24 = 0$$

$$(y+6)(y+4) = 0$$

$$y = -6 \text{ or } -4$$

$$\therefore x = -2, y = -6$$

$$\text{or } x = 2, y = -4$$

n  $y = \frac{3}{2}x - 2$

sub.

$$2x^2 + x - 4(\frac{3}{2}x - 2) = 6$$

$$2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$$x = \frac{1}{2} \text{ or } 2$$

$$\therefore x = \frac{1}{2}, y = -\frac{5}{4}$$

$$\text{or } x = 2, y = 1$$

o  $x = 3y - 17$

sub.

$$(3y-17)^2 + (3y-17) + 2y^2 - 52 = 0$$

$$y^2 - 9y + 20 = 0$$

$$(y-4)(y-5) = 0$$

$$y = 4 \text{ or } 5$$

$$\therefore x = -5, y = 4$$

$$\text{or } x = -2, y = 5$$

4 a subtracting

$$-\frac{1}{y} + 2y + 1 = 0$$

$$-1 + 2y^2 + y = 0$$

$$2y^2 + y - 1 = 0$$

$$(2y-1)(y+1) = 0$$

$$y = -1 \text{ or } \frac{1}{2}$$

$$\therefore x = -5, y = -1$$

$$\text{or } x = 4, y = \frac{1}{2}$$

b  $y = x - 5$

sub.

$$x(x-5) = 6$$

$$x^2 - 5x - 6 = 0$$

$$(x+1)(x-6) = 0$$

$$x = -1 \text{ or } 6$$

$$\therefore x = -1, y = -6$$

$$\text{or } x = 6, y = 1$$

c  $y = 7 - 4x$

sub.

$$\frac{3}{x} - 2(7 - 4x) + 4 = 0$$

$$3 - 2x(7 - 4x) + 4x = 0$$

$$8x^2 - 10x + 3 = 0$$

$$(4x-3)(2x-1) = 0$$

$$x = \frac{1}{2} \text{ or } \frac{3}{4}$$

$$\therefore x = \frac{1}{2}, y = 5$$

$$\text{or } x = \frac{3}{4}, y = 4$$

5  $5 - x = x^2 - 3x + 2$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1 \text{ or } 3$$

P and Q are the points  $(-1, 6)$  and  $(3, 2)$

$$PQ^2 = (3+1)^2 + (2-6)^2$$

$$PQ = \sqrt{32} = 4\sqrt{2}$$

6  $3^{x-1} = (3^2)^{2y}$

$$(2^3)^{x-2} = (2^2)^{1+y}$$

$$\therefore x-1 = 4y$$

$$\therefore 3x-6 = 2+2y$$

$$6x-16 = 4y$$

$$\Rightarrow 6x-16 = x-1$$

$$x = 3$$

$$\therefore x = 3, y = \frac{1}{2}$$

7  $AB - A\sqrt{3} + 2B\sqrt{3} - 6 \equiv 9\sqrt{3} - 1$

A and B integers  $\therefore AB - 6 = -1 \quad (1)$  and  $-A + 2B = 9 \quad (2)$

$$(2) \Rightarrow A = 2B - 9$$

$$\text{sub. (1)} \quad B(2B-9)-6=-1 \quad \Rightarrow \quad 2B^2 - 9B - 5 = 0$$

$$(2B+1)(B-5) = 0$$

$$B = -\frac{1}{2} \text{ or } 5$$

$$B \text{ integer } \therefore B = 5 \quad \Rightarrow \quad A = 1, B = 5$$