1 **a** $= (\frac{3}{2})^2 = \frac{9}{4}$ or $2\frac{1}{4}$

b $x^{\frac{3}{2}} = 27$

$x^{\frac{1}{2}} = \sqrt[3]{27} = 3$

$x = 3^2 = 9$

2 $x = 16 - 3y$

sub. $(16 - 3y)^2 - y(16 - 3y) + 2y^2 = 46$

$y^2 - 8y + 15 = 0$

$(y - 3)(y - 5) = 0$

$y = 3$ or 5

$\therefore\ x = 1, y = 5$ or $x = 7, y = 3$

3 **a** $= 8\sqrt{3} - 4\sqrt{3} + 5\sqrt{3}$

$= 9\sqrt{3}$

b $= 10 - 4\sqrt{3} + 5\sqrt{3} - 6$

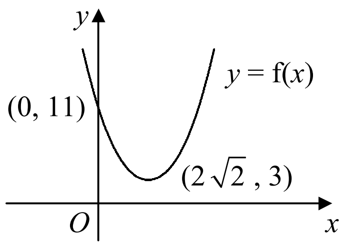
$= 4 + \sqrt{3}$

4 **a** $\text{f}(x) = (x - 2\sqrt{2})^2 - 8 + 11$

$= (x - 2\sqrt{2})^2 + 3$

$\therefore\ a = 1, b = -2\sqrt{2}$ and $c = 3$

b turning point is $(2\sqrt{2}, 3)$



5 **a** $\text{S.A} = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$

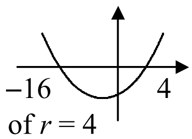
$\text{S.A} \le 128\pi\ \therefore\ 2\pi r^2 + 24\pi r \le 128\pi$

$r^2 + 12r \le 64$

$r^2 + 12r - 64 \le 0$

b $(r + 16)(r - 4) \le 0$

$-16 \le r \le 4$



$\therefore$ maximum value of $r = 4$

6 $8x\sqrt{x} = 4x$

$4x(2\sqrt{x} - 1) = 0$

$x \ne 0\ \therefore\ \sqrt{x} = \frac{1}{2}$

$x = (\frac{1}{2})^2 = \frac{1}{4}$

7 **a** $x = 5$

b $(2^5)^{y+1} = (2^2)^y$

$5y + 5 = 2y$

$y = -\frac{5}{3}$

8 **a** $t^2 - 5t$

b $t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t = 2, 3$

$x = t^2 = 4, 9$

9 $x^2 + kx + 3 + k^2 = 0$

$\Rightarrow\quad (x + \frac{1}{2}k)^2 - \frac{1}{4}k^2 + 3 + k^2 = 0$

$\Rightarrow\quad x + \frac{1}{2}k = \pm\sqrt{-\frac{3}{4}k^2 - 3}$

$\Rightarrow\quad x = -\frac{1}{2}k \pm \sqrt{-\frac{3}{4}k^2 - 3}$

real $k \Rightarrow k^2 \ge 0$

$\Rightarrow\ -\frac{3}{4}k^2 - 3 < 0$

$\therefore$ no real roots

10 **a** $(2^3)^{2x-1} = 2^5$

$6x - 3 = 5$

$x = \frac{4}{3}$

b $(3^{-1})^{y-2} = 3^4$

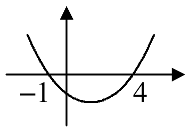
$-y + 2 = 4$

$y = -2$

11 $2x^2 - 7x < x^2 - 4x + 4$

$x^2 - 3x - 4 < 0$

$(x + 1)(x - 4) < 0$



$-1 < x < 4$

12 $\frac{2}{3\sqrt{2}-4} = \frac{2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{2(3\sqrt{2}+4)}{18-16} = 3\sqrt{2} + 4$

$\frac{3-\sqrt{2}}{\sqrt{2}+1} = \frac{3-\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(3-\sqrt{2})(\sqrt{2}-1)}{2-1}$

$= (3 - \sqrt{2})(\sqrt{2} - 1)$

$= 3\sqrt{2} - 3 - 2 + \sqrt{2} = 4\sqrt{2} - 5$

$\therefore \frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1} = 3\sqrt{2} + 4 - (4\sqrt{2} - 5)$

$= 9 - \sqrt{2}$

13 **a** $(3y - 1)(2y + 9) = 0$

$y = -\frac{9}{2}$ or $\frac{1}{3}$

b equal roots

$\therefore b^2 - 4ac = 0$

$k^2 - 64 = 0$

$k = \pm 8$

14 **a** **i** $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

ii $2^{x-1} = 2^{-1} \times 2^x = \frac{1}{2}y$

b let $y = 2^x \Rightarrow y^2 - 9(\frac{1}{2}y) + 2 = 0$

$2y^2 - 9y + 4 = 0$

$(2y - 1)(y - 4) = 0$

$y = 2^x = \frac{1}{2}$ or 4

$x = -1$ or 2

15 $x = 3y + 1$

sub.

$(3y + 1)^2 + 2y(3y + 1) + y^2 = 9$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = -1$ or $\frac{1}{2}$

$\therefore (-2, -1)$ and $(\frac{5}{2}, \frac{1}{2})$

16 **a** $(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + b = 0$

$(x + \frac{1}{2}a)^2 = \frac{1}{4}a^2 - b = \frac{a^2 - 4b}{4}$

$x + \frac{1}{2}a = \pm\sqrt{\frac{a^2 - 4b}{4}} = \pm\frac{\sqrt{a^2 - 4b}}{2}$

$x = -\frac{1}{2}a \pm \frac{\sqrt{a^2 - 4b}}{2}$

$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

b for repeated root, $a^2 - 4b = 0$

$\Rightarrow b = \frac{1}{4}a^2$

17 **a** $\text{f}(x) = 2[x^2 - 6x] + 19$

$= 2[(x - 3)^2 - 9] + 19$

$= 2(x - 3)^2 + 1$

real $x \Rightarrow (x - 3)^2 \geq 0$

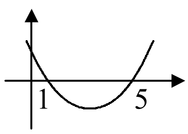
$\Rightarrow 2(x - 3)^2 + 1 \geq 1$

$\Rightarrow \text{f}(x) \geq 1$

b $2x^2 - 12x + 19 < 9$

$x^2 - 6x + 5 < 0$

$(x - 1)(x - 5) < 0$



$1 < x < 5$

18 **a** $= 1 - 2\sqrt{5} + 5$

$= 6 - 2\sqrt{5}$

b $y^2 = \frac{1}{2}(6 - 2\sqrt{5}) = \frac{1}{2}(1 - \sqrt{5})^2$

$y = \pm\frac{1}{\sqrt{2}}(1 - \sqrt{5})$

$y = \pm\frac{1}{2}\sqrt{2}(1 - \sqrt{5})$

$y = \pm\frac{1}{2}(\sqrt{2} - \sqrt{10})$

$y = \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{10}$ or $-\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{10}$