1 **a**

$$\begin{array}{r} x^2 + x - 2 \\ x+1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \underline{x^3 + x^2} \\ x^2 - x \\ \underline{x^2 + x} \\ -2x - 2 \\ \underline{-2x - 2} \end{array}$$

quotient: $x^2 + x - 2$

b

$$\begin{array}{r} x^2 + 4x - 1 \\ x-2 \overline{)\, x^3 + 2x^2 - 9x + 2} \\ \underline{x^3 - 2x^2} \\ 4x^2 - 9x \\ \underline{4x^2 - 8x} \\ -x + 2 \\ \underline{-x + 2} \end{array}$$

quotient: $x^2 + 4x - 1$

c

$$\begin{array}{r} x^2 - x + 5 \\ x+4 \overline{)\, x^3 + 3x^2 + x + 20} \\ \underline{x^3 + 4x^2} \\ -x^2 + x \\ \underline{-x^2 - 4x} \\ 5x + 20 \\ \underline{5x + 20} \end{array}$$

quotient: $x^2 - x + 5$

d

$$\begin{array}{r} 2x^2 + x - 3 \\ x-1 \overline{)\, 2x^3 - x^2 - 4x + 3} \\ \underline{2x^3 - 2x^2} \\ x^2 - 4x \\ \underline{x^2 - x} \\ -3x + 3 \\ \underline{-3x + 3} \end{array}$$

quotient: $2x^2 + x - 3$

e

$$\begin{array}{r} 6x^2 + 11x - 18 \\ x-5 \overline{)\, 6x^3 - 19x^2 - 73x + 90} \\ \underline{6x^3 - 30x^2} \\ 11x^2 - 73x \\ \underline{11x^2 - 55x} \\ -18x + 90 \\ \underline{-18x + 90} \end{array}$$

quotient: $6x^2 + 11x - 18$

f

$$\begin{array}{r} -x^2 + 7x - 4 \\ x+2 \overline{)\, -x^3 + 5x^2 + 10x - 8} \\ \underline{-x^3 - 2x^2} \\ 7x^2 + 10x \\ \underline{7x^2 + 14x} \\ -4x - 8 \\ \underline{-4x - 8} \end{array}$$

quotient: $-x^2 + 7x - 4$

g

$$\begin{array}{r} x^2 - 3x + 7 \\ x+3 \overline{)\, x^3 + 0x^2 - 2x + 21} \\ \underline{x^3 + 3x^2} \\ -3x^2 - 2x \\ \underline{-3x^2 - 9x} \\ 7x + 21 \\ \underline{7x + 21} \end{array}$$

quotient: $x^2 - 3x + 7$

h

$$\begin{array}{r} 3x^2 - 2x + 12 \\ x+6 \overline{)\, 3x^3 + 16x^2 + 0x + 72} \\ \underline{3x^3 + 18x^2} \\ -2x^2 + 0x \\ \underline{-2x^2 - 12x} \\ 12x + 72 \\ \underline{12x + 72} \end{array}$$

quotient: $3x^2 - 2x + 12$

2 **a**

$$\begin{array}{r} x^2 + 3x + 2 \\ x+5 \overline{)\, x^3 + 8x^2 + 17x + 16} \\ x^3 + 5x^2 \\ \hline 3x^2 + 17x \\ 3x^2 + 15x \\ \hline 2x + 16 \\ 2x + 10 \\ \hline 6 \end{array}$$

quotient: $x^2 + 3x + 2$ remainder: 6

b

$$\begin{array}{r} x^2 - 8x + 5 \\ x-7 \overline{)\, x^3 - 15x^2 + 61x - 48} \\ x^3 - 7x^2 \\ \hline -8x^2 + 61x \\ -8x^2 + 56x \\ \hline 5x - 48 \\ 5x - 35 \\ \hline -13 \end{array}$$

quotient: $x^2 - 8x + 5$ remainder: -13

c

$$\begin{array}{r} 3x^2 - 2x + 4 \\ x+2 \overline{)\, 3x^3 + 4x^2 + 0x + 7} \\ 3x^3 + 6x^2 \\ \hline -2x^2 + 0x \\ -2x^2 - 4x \\ \hline 4x + 7 \\ 4x + 8 \\ \hline -1 \end{array}$$

quotient: $3x^2 - 2x + 4$ remainder: -1

d

$$\begin{array}{r} -x^2 + 3x - 9 \\ x+8 \overline{)\, -x^3 - 5x^2 + 15x - 50} \\ -x^3 - 8x^2 \\ \hline 3x^2 + 15x \\ 3x^2 + 24x \\ \hline -9x - 50 \\ -9x - 72 \\ \hline 22 \end{array}$$

quotient: $-x^2 + 3x - 9$ remainder: 22

e

$$\begin{array}{r} 4x^2 + 14x + 26 \\ x-3 \overline{)\, 4x^3 + 2x^2 - 16x + 3} \\ 4x^3 - 12x^2 \\ \hline 14x^2 - 16x \\ 14x^2 - 42x \\ \hline 26x + 3 \\ 26x - 78 \\ \hline 81 \end{array}$$

quotient: $4x^2 + 14x + 26$ remainder: 81

f

$$\begin{array}{r} -6x^2 - 10x + 20 \\ x+2 \overline{)\, -6x^3 - 22x^2 + 0x + 1} \\ -6x^3 - 12x^2 \\ \hline -10x^2 + 0x \\ -10x^2 - 20x \\ \hline 20x + 1 \\ 20x + 40 \\ \hline -39 \end{array}$$

quotient: $-6x^2 - 10x + 20$ remainder: -39

3 **a** let $f(x) \equiv x^3 + 2x^2 - 2x - 1$
$f(1) = 1 + 2 - 2 - 1 = 0$
$\therefore (x - 1)$ is a factor

b let $f(x) \equiv x^3 - 5x^2 - 9x + 2$
$f(-2) = -8 - 20 + 18 + 2 = -8$
$\therefore (x + 2)$ is not a factor

c let $f(x) \equiv x^3 - x^2 - 14x + 27$
$f(3) = 27 - 9 - 42 + 27 = 3$
$\therefore (x - 3)$ is not a factor

d let $f(x) \equiv 2x^3 + 13x^2 + 2x - 24$
$f(-6) = -432 + 468 - 12 - 24 = 0$
$\therefore (x + 6)$ is a factor

e let $f(x) \equiv 2x^3 - 5x^2 + 7x - 14$
$f(-\frac{1}{2}) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$
$\therefore (2x + 1)$ is not a factor

f let $f(x) \equiv 2 - 17x + 25x^2 - 6x^3$
$f(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$
$\therefore (3x - 2)$ is a factor

4 **a** $f(1) = 1 - 2 - 11 + 12 = 0$
$\therefore (x - 1)$ is a factor of $f(x)$

b

$$\begin{array}{r} x^2 - x - 12 \\ x-1 \overline{)\, x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2} \\ -x^2 - 11x \\ \underline{-x^2 + x} \\ -12x + 12 \\ \underline{-12x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 1)(x^2 - x - 12)$
$\equiv (x - 1)(x + 3)(x - 4)$

5 $g(-3) = -54 + 9 + 39 + 6 = 0$
$\therefore (x + 3)$ is a factor of $g(x)$

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{)\, 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \\ -5x^2 - 13x \\ \underline{-5x^2 - 15x} \\ 2x + 6 \\ \underline{2x + 6} \end{array}$$

$\therefore g(x) \equiv (x + 3)(2x^2 - 5x + 2)$
$\equiv (x + 3)(2x - 1)(x - 2)$

$g(x) = 0 \Rightarrow (x + 3)(2x - 1)(x - 2) = 0$
$x = -3, \frac{1}{2}$ or 2

6 $f(4) = 0 \quad \therefore (x - 4)$ is a factor of $f(x)$

$$\begin{array}{r} 6x^2 + 17x - 3 \\ x-4 \overline{)\, 6x^3 - 7x^2 - 71x + 12} \\ \underline{6x^3 - 24x^2} \\ 17x^2 - 71x \\ \underline{17x^2 - 68x} \\ -3x + 12 \\ \underline{-3x + 12} \end{array}$$

$\therefore f(x) \equiv (x - 4)(6x^2 + 17x - 3)$
$\equiv (x - 4)(6x - 1)(x + 3)$

$f(x) = 0 \Rightarrow (x - 4)(6x - 1)(x + 3) = 0$

$x = -3, \frac{1}{6}$ or 4

7 **a** $g(-2) = 0 \quad \therefore (x + 2)$ is a factor of $g(x)$

$$\begin{array}{r} x^2 + 5x - 3 \\ x+2 \overline{)\, x^3 + 7x^2 + 7x - 6} \\ \underline{x^3 + 2x^2} \\ 5x^2 + 7x \\ \underline{5x^2 + 10x} \\ -3x - 6 \\ \underline{-3x - 6} \end{array}$$

$\therefore g(x) \equiv (x + 2)(x^2 + 5x - 3)$

b other solutions given by $x^2 + 5x - 3 = 0$

$x = \frac{-5 \pm \sqrt{25 + 12}}{2} = \frac{-5 \pm \sqrt{37}}{2}$

$x = -5.54$ or 0.54

8 **a** $f(1) = 1 + 2 - 11 - 12 = -20$
$f(2) = 8 + 8 - 22 - 12 = -18$
$f(-1) = -1 + 2 + 11 - 12 = 0$
$f(-2) = -8 + 8 + 22 - 12 = 10$

b $(x + 1)$ is a factor of $f(x)$

$$\begin{array}{r} x^2 + x - 12 \\ x+1 \overline{)\, x^3 + 2x^2 - 11x - 12} \\ \underline{x^3 + x^2} \\ x^2 - 11x \\ \underline{x^2 + x} \\ -12x - 12 \\ \underline{-12x - 12} \end{array}$$

$\therefore f(x) = (x + 1)(x^2 + x - 12)$
$= (x + 1)(x + 4)(x - 3)$

9 **a** let $f(x) = x^3 - 2x^2 - 5x + 6$
$f(1) = 0$
$\therefore (x - 1)$ is a factor

$$\begin{array}{r|l} x-1 & x^2 - x - 6 \\ \hline & x^3 - 2x^2 - 5x + 6 \\ & x^3 - x^2 \\ & \hline & -x^2 - 5x \\ & -x^2 + x \\ & \hline & -6x + 6 \\ & -6x + 6 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x - 1)(x^2 - x - 6)$
$= (x - 1)(x + 2)(x - 3)$

b let $f(x) = x^3 + x^2 - 5x - 2$
$f(1) = -5$, $f(2) = 0$
$\therefore (x - 2)$ is a factor

$$\begin{array}{r|l} x-2 & x^2 + 3x + 1 \\ \hline & x^3 + x^2 - 5x - 2 \\ & x^3 - 2x^2 \\ & \hline & 3x^2 - 5x \\ & 3x^2 - 6x \\ & \hline & x - 2 \\ & x - 2 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x - 2)(x^2 + 3x + 1)$

c let $f(x) = 20 + 11x - 8x^2 + x^3$
$f(1) = 24$, $f(2) = 18$, $f(-1) = 0$
$\therefore (x + 1)$ is a factor

$$\begin{array}{r|l} x+1 & x^2 - 9x + 20 \\ \hline & x^3 - 8x^2 + 11x + 20 \\ & x^3 + x^2 \\ & \hline & -9x^2 + 11x \\ & -9x^2 - 9x \\ & \hline & 20x + 20 \\ & 20x + 20 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x + 1)(x^2 - 9x + 20)$
$= (x + 1)(x - 4)(x - 5)$

d let $f(x) = 3x^3 - 4x^2 - 35x + 12$
$f(1) = -24$, $f(2) = -50$,
$f(-1) = 40$, $f(-2) = 42$
$f(3) = -48$, $f(-3) = 0$
$\therefore (x + 3)$ is a factor

$$\begin{array}{r|l} x+3 & 3x^2 - 13x + 4 \\ \hline & 3x^3 - 4x^2 - 35x + 12 \\ & 3x^3 + 9x^2 \\ & \hline & -13x^2 - 35x \\ & -13x^2 - 39x \\ & \hline & 4x + 12 \\ & 4x + 12 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x + 3)(3x^2 - 13x + 4)$
$= (x + 3)(3x - 1)(x - 4)$

e let $f(x) = x^3 + 8$
$f(1) = 9$, $f(2) = 16$
$f(-1) = 7$, $f(-2) = 0$
$\therefore (x + 2)$ is a factor

$$\begin{array}{r|l} x+2 & x^2 - 2x + 4 \\ \hline & x^3 + 0x^2 + 0x + 8 \\ & x^3 + 2x^2 \\ & \hline & -2x^2 + 0x \\ & -2x^2 - 4x \\ & \hline & 4x + 8 \\ & 4x + 8 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x + 2)(x^2 - 2x + 4)$

f let $f(x) = 12 + 29x + 8x^2 - 4x^3$
$f(1) = 45$, $f(2) = 70$,
$f(-1) = -5$, $f(-2) = 18$
$f(3) = 63$, $f(-3) = 105$
$f(4) = 0$
$\therefore (x - 4)$ is a factor

$$\begin{array}{r|l} x-4 & -4x^2 - 8x - 3 \\ \hline & -4x^3 + 8x^2 + 29x + 12 \\ & -4x^3 + 16x^2 \\ & \hline & -8x^2 + 29x \\ & -8x^2 + 32x \\ & \hline & -3x + 12 \\ & -3x + 12 \\ & \hline \end{array}$$

$\therefore$
$f(x) = (x - 4)(-4x^2 - 8x - 3)$
$= -(x - 4)(4x^2 + 8x + 3)$
$= (4 - x)(2x + 1)(2x + 3)$

10 **a** let $f(x) = x^3 - x^2 - 10x - 8$
$f(1) = -18$, $f(2) = -24$,
$f(-1) = 0$
$\therefore (x + 1)$ is a factor

$$\begin{array}{r} x^2 - 2x - 8 \\ x+1 \overline{)\, x^3 - x^2 - 10x - 8} \\ x^3 + x^2 \\ \hline -2x^2 - 10x \\ -2x^2 - 2x \\ \hline -8x - 8 \\ -8x - 8 \\ \hline \end{array}$$

$\therefore$
$(x + 1)(x^2 - 2x - 8) = 0$
$(x + 1)(x + 2)(x - 4) = 0$
$x = -2, -1, 4$

b let $f(x) = x^3 + 2x^2 - 9x - 18$
$f(1) = -24$, $f(2) = -20$
$f(-1) = -8$, $f(-2) = 0$
$\therefore (x + 2)$ is a factor

$$\begin{array}{r} x^2 + 0x - 9 \\ x+2 \overline{)\, x^3 + 2x^2 - 9x - 18} \\ x^3 + 2x^2 \\ \hline 0x^2 - 9x \\ 0x^2 + 0x \\ \hline -9x - 18 \\ -9x - 18 \\ \hline \end{array}$$

$\therefore$
$(x + 2)(x^2 - 9) = 0$
$(x + 2)(x + 3)(x - 3) = 0$
$x = -3, -2, 3$

c let $f(x) = 4x^3 - 12x^2 + 9x - 2$
$f(1) = -1$, $f(2) = 0$
$\therefore (x - 2)$ is a factor

$$\begin{array}{r} 4x^2 - 4x + 1 \\ x-2 \overline{)\, 4x^3 - 12x^2 + 9x - 2} \\ 4x^3 - 8x^2 \\ \hline -4x^2 + 9x \\ -4x^2 + 8x \\ \hline x - 2 \\ x - 2 \\ \hline \end{array}$$

$\therefore$
$(x - 2)(4x^2 - 4x + 1) = 0$
$(x - 2)(2x - 1)^2 = 0$
$x = \frac{1}{2}, 2$

d let $f(x) = x^3 - 5x^2 + 3x + 1$
$f(1) = 0$
$\therefore (x - 1)$ is a factor

$$\begin{array}{r} x^2 - 4x - 1 \\ x-1 \overline{)\, x^3 - 5x^2 + 3x + 1} \\ x^3 - x^2 \\ \hline -4x^2 + 3x \\ -4x^2 + 4x \\ \hline -x + 1 \\ -x + 1 \\ \hline \end{array}$$

$\therefore$
$(x - 1)(x^2 - 4x - 1) = 0$
$x = 1$ or $\frac{4 \pm \sqrt{16 + 4}}{2}$
$x = 1, 2 \pm \sqrt{5}$

e let $f(x) = x^3 + 4x^2 - 9x - 6$
$f(1) = -10$, $f(2) = 0$
$\therefore (x - 2)$ is a factor

$$\begin{array}{r} x^2 + 6x + 3 \\ x-2 \overline{)\, x^3 + 4x^2 - 9x - 6} \\ x^3 - 2x^2 \\ \hline 6x^2 - 9x \\ 6x^2 - 12x \\ \hline 3x - 6 \\ 3x - 6 \\ \hline \end{array}$$

$\therefore$
$(x - 2)(x^2 + 6x + 3) = 0$
$x = 2$ or $\frac{-6 \pm \sqrt{36 - 12}}{2}$
$x = 2, -3 \pm \sqrt{6}$

f let $f(x) = x^3 - 14x + 15$
$f(1) = 2$, $f(2) = -5$, $f(-1) = 28$,
$f(-2) = 35$, $f(3) = 0$
$\therefore (x - 3)$ is a factor

$$\begin{array}{r} x^2 + 3x - 5 \\ x-3 \overline{)\, x^3 + 0x^2 - 14x + 15} \\ x^3 - 3x^2 \\ \hline 3x^2 - 14x \\ 3x^2 - 9x \\ \hline -5x + 15 \\ -5x + 15 \\ \hline \end{array}$$

$\therefore$
$(x - 3)(x^2 + 3x - 5) = 0$
$x = 3$ or $\frac{-3 \pm \sqrt{9 + 20}}{2}$
$x = 3, \frac{1}{2}(-3 \pm \sqrt{29})$

11 **a** $f(2) = 0$
$\therefore 16 - 4 - 30 + c = 0$
$c = 18$

b

$$\begin{array}{r} 2x^2 + 3x - 9 \\ x-2 \overline{)\, 2x^3 - x^2 - 15x + 18} \\ 2x^3 - 4x^2 \\ \hline 3x^2 - 15x \\ 3x^2 - 6x \\ \hline -9x + 18 \\ -9x + 18 \\ \hline \end{array}$$

$\therefore f(x) \equiv (x - 2)(2x^2 + 3x - 9)$
$\equiv (x - 2)(2x - 3)(x + 3)$

12 **a** $g(-1) = 0$
$\therefore -1 + p + 13 + q = 0$
$p + q + 12 = 0$ (1)
$g(3) = 0$
$\therefore 27 + 9p - 39 + q = 0$
$9p + q - 12 = 0$ (2)
(2) − (1) $\Rightarrow 8p - 24 = 0 \Rightarrow p = 3$
sub (1) $\Rightarrow 3 + q + 12 = 0 \Rightarrow q = -15$

b $(x + 1)(x - 3)(ax + b) \equiv x^3 + 3x^2 - 13x - 15$
by inspection
$g(x) \equiv (x + 1)(x - 3)(x + 5)$
$g(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 5) = 0$
$x = -5, -1$ or 3

13 **a** $= f(2) = 8 + 16 - 2 + 6 = 28$

b $= f(-1) = -1 - 2 - 7 + 1 = -9$

c $= f(-5) = -250 + 25 - 45 + 17 = -163$

d $= f(\frac{1}{2}) = 1 + 1 - 3 - 3 = -4$

e $= f(-\frac{1}{2}) = -\frac{1}{4} - \frac{3}{4} + 10 - 7 = 2$

f $= f(\frac{2}{3}) = \frac{8}{9} - \frac{8}{3} + \frac{4}{3} - 7 = -7\frac{4}{9}$

14 $f(2) = 5$

$\therefore\ 8 - 16 + 10 + c = 5$

$c = 3$

15 $f(\frac{1}{2}) = -2$

$\therefore\ \frac{1}{4} - \frac{9}{4} + \frac{1}{2}k + 5 = -2$

$k = -10$

16 **a** $f(-3) = 22$

$\therefore\ -54 + 9a + 13 = 22$

$a = 7$

b $f(x) = 2x^3 + 7x^2 + 13$

remainder $= f(4)$

$= 128 + 112 + 13$

$= 253$

17 **a** $f(-1) = 0$

$\therefore\ -p + q - q + 3 = 0$

$p = 3$

b $f(x) = 3x^3 + qx^2 + qx + 3$

$f(2) = 15$

$\therefore\ 24 + 4q + 2q + 3 = 15$

$q = -2$

18 **a** $p(3) = 0$

$\therefore\ 27 + 9a + 27 + b = 0$

$9a + b = -54 \quad (1)$

b $p(-2) = -30$

$\therefore\ -8 + 4a - 18 + b = -30$

$4a + b = -4 \quad (2)$

$(1) - (2) \Rightarrow \quad 5a = -50$

$\therefore\ a = -10,\ b = 36$

19 $f(-1) = 3$

$\therefore\ -4 - 6 - m + n = 3$

$n - m = 13 \quad (1)$

$f(\frac{1}{2}) = 15$

$\therefore\ \frac{1}{2} - \frac{3}{2} + \frac{1}{2}m + n = 15$

$n + \frac{1}{2}m = 16 \quad (2)$

$(2) - (1) \Rightarrow \quad \frac{3}{2}m = 3$

$\therefore\ m = 2,\ n = 15$

20 **a** $g(4) = 39$

$\therefore\ 64 + 4c + 3 = 39$

$c = -7$

b $g(x) = x^3 - 7x + 3$

$$
\begin{array}{r|l}
x + 2 & x^2 - 2x - 3 \\
\hline
 & x^3 + 0x^2 - 7x + 3 \\
 & x^3 + 2x^2 \\
\hline
 & \quad -2x^2 - 7x \\
 & \quad -2x^2 - 4x \\
\hline
 & \qquad -3x + 3 \\
 & \qquad -3x - 6 \\
\hline
 & \qquad\quad 9
\end{array}
$$

quotient $= x^2 - 2x - 3$

remainder $= 9$