**1** **a** $f(-2) = 0 \Rightarrow -8 - 20 - 2a + b = 0$

$\Rightarrow -2a + b = 28 \quad (1)$

$f(3) = 0 \Rightarrow 27 - 45 + 3a + b = 0$

$\Rightarrow 3a + b = 18 \quad (2)$

$(2) - (1) \;\; 5a = -10 = 0 \Rightarrow a = -2$

sub. (1) $\Rightarrow b = 24$

**b** $f(x) \equiv x^3 - 5x^2 - 2x + 24$

$(x + 2)(x - 3)(ax + b) \equiv x^3 - 5x^2 - 2x + 24$

by inspection

$f(x) \equiv (x + 2)(x - 3)(x - 4)$

**2** $f(k) = 8f(\tfrac{1}{2}k)$

$8k^3 - k^2 + 7 = 8(k^3 - \tfrac{1}{4}k^2 + 7)$

$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$

$k^2 = 49$

$k = \pm 7$

**3** **a** $f(2) = 24 - 4 - 24 + 4 = 0$

$\therefore (x - 2)$ is a factor of $f(x)$

**b**

$$\begin{array}{r|l} & 3x^2 + 5x - 2 \\ \hline x - 2 & 3x^3 - x^2 - 12x + 4 \\ & 3x^3 - 6x^2 \\ \hline & 5x^2 - 12x \\ & 5x^2 - 10x \\ \hline & -2x + 4 \\ & -2x + 4 \\ \hline \end{array}$$

$\therefore f(x) = (x - 2)(3x^2 + 5x - 2)$

$= (x - 2)(3x - 1)(x + 2)$

$f(x) = 0 \Rightarrow (x - 2)(3x - 1)(x + 2) = 0$

$x = -2, \tfrac{1}{3}$ or $2$

**4** $6 + 7x - x^3 = 0$

let $f(x) = 6 + 7x - x^3$

$f(1) = 12, \; f(2) = 12, \; f(-1) = 0$

$\therefore (x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & -x^2 + x + 6 \\ \hline x + 1 & -x^3 + 0x^2 + 7x + 6 \\ & -x^3 - x^2 \\ \hline & x^2 + 7x \\ & x^2 + x \\ \hline & 6x + 6 \\ & 6x + 6 \\ \hline \end{array}$$

$\therefore (x + 1)(-x^2 + x + 6) = 0$

$-(x + 1)(x - 3)(x + 2) = 0$

$x = -2, -1, 3$

$\therefore (-2, 0), (-1, 0)$ and $(3, 0)$

**5** **a** $f(-1) = -4$

$\therefore\ -3 + p - 8 + q = -4$

$p + q = 7 \qquad (1)$

$f(2) = 80$

$\therefore\ 24 + 4p + 16 + q = 80$

$4p + q = 40 \qquad (2)$

$(2) - (1) \Rightarrow 3p = 33$

$\therefore\ p = 11,\ q = -4$

**b** $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore\ (x + 2)$ is a factor

**c**

$$\begin{array}{r|rrrr}
 & 3x^2 & +\ 5x & -\ 2 & \\
\hline
x+2 & 3x^3 & +\ 11x^2 & +\ 8x & -\ 4 \\
 & 3x^3 & +\ 6x^2 & & \\
\hline
 & & 5x^2 & +\ 8x & \\
 & & 5x^2 & +\ 10x & \\
\hline
 & & & -\ 2x & -\ 4 \\
 & & & -\ 2x & -\ 4 \\
\hline
\end{array}$$

$\therefore\ f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

$\therefore\ f(x) = 0 \Rightarrow x = -2$ or $\frac{1}{3}$

**6** **a** let $f(x) = x^3 - 4x^2 - 7x + 10$

$f(1) = 1 - 4 - 7 + 10 = 0$

$\therefore\ (x - 1)$ is a factor

$$\begin{array}{r|rrrr}
 & x^2 & -\ 3x & -\ 10 & \\
\hline
x-1 & x^3 & -\ 4x^2 & -\ 7x & +\ 10 \\
 & x^3 & -\ x^2 & & \\
\hline
 & & -\ 3x^2 & -\ 7x & \\
 & & -\ 3x^2 & +\ 3x & \\
\hline
 & & & -\ 10x & +\ 10 \\
 & & & -\ 10x & +\ 10 \\
\hline
\end{array}$$

$\therefore\ (x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

$x = -2, 1, 5$

**b** $y^2 = x$ in part **a**

$y^2 = 1, 5$ or $-2$ [no solutions]

$y = \pm 1, \pm\sqrt{5}$

**7** **a** $= f(-1) = -1 + 7 - 14 + 3 = -5$

**b**

$$\begin{array}{r|rrrr}
 & n^2 & +\ 6n & +\ 8 & \\
\hline
n+1 & n^3 & +\ 7n^2 & +\ 14n & +\ 3 \\
 & n^3 & +\ n^2 & & \\
\hline
 & & 6n^2 & +\ 14n & \\
 & & 6n^2 & +\ 6n & \\
\hline
 & & & 8n & +\ 3 \\
 & & & 8n & +\ 8 \\
\hline
 & & & & -\ 5
\end{array}$$

$\therefore\ f(n) = (n + 1)(n^2 + 6n + 8) - 5$

$f(n) = (n + 1)(n + 2)(n + 4) - 5$

**c** $(n + 1)$ and $(n + 2)$ are consecutive integers

$\therefore$ either $(n + 1)$ or $(n + 2)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4) - 5$ is odd