1 $$f(x) \equiv 2x^3 + 5x^2 - 1.$$

a Find $f'(x)$.

b Find the set of values of x for which $f(x)$ is increasing.

2 The curve C has the equation $y = x^3 - x^2 + 2x - 4$.

a Find an equation of the tangent to C at the point $(1, -2)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Prove that the curve C has no stationary points.

3 A curve has the equation $y = \sqrt{x} + \frac{4}{x}$.

a Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve and determine its nature.

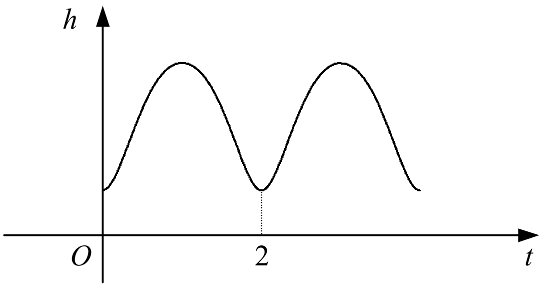
4 $$f(x) \equiv x^3 + 6x^2 + 9x.$$

a Find the coordinates of the points where the curve $y = f(x)$ meets the x-axis.

b Find the set of values of x for which $f(x)$ is decreasing.

c Sketch the curve $y = f(x)$, showing the coordinates of any stationary points.

5



The graph shows the height, h cm, of the letters on a website advert t seconds after the advert appears on the screen.

For t in the interval $0 \leq t \leq 2$, h is given by the equation

$$h = 2t^4 - 8t^3 + 8t^2 + 1.$$

For larger values of t, the variation of h over this interval is repeated every 2 seconds.

a Find $\frac{dh}{dt}$ for t in the interval $0 \leq t \leq 2$.

b Find the rate at which the height of the letters is increasing when $t = 0.25$

c Find the maximum height of the letters.

6 The curve C has the equation $y = x^3 + 3kx^2 - 9k^2x$, where k is a non-zero constant.

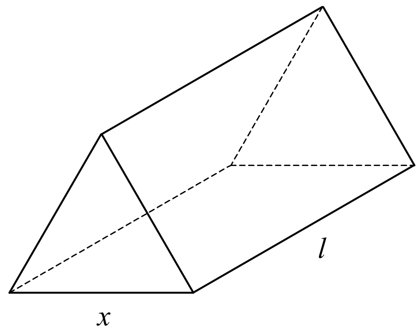
a Show that C is stationary when

$$x^2 + 2kx - 3k^2 = 0.$$

b Hence, show that C is stationary at the point with coordinates $(k, -5k^3)$.

c Find, in terms of k, the coordinates of the other stationary point on C.

7



The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is l cm.

Given that the volume of the prism is 250 cm^3,

a find an expression for l in terms of x,

b show that the surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x}).$$

Given that x can vary,

c find the value of x for which A is a minimum,

d find the minimum value of A in the form $k\sqrt{3}$,

e justify that the value you have found is a minimum.

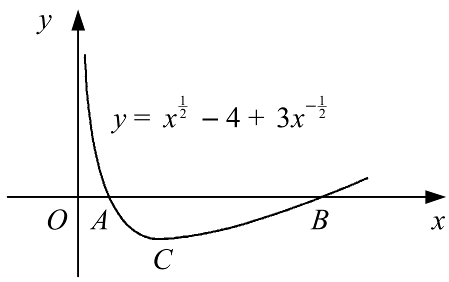
8 $$\text{f}(x) \equiv x^3 + 4x^2 + kx + 1.$$

a Find the set of values of the constant k for which the curve $y = \text{f}(x)$ has two stationary points.

Given that $k = -3$,

b find the coordinates of the stationary points of the curve $y = \text{f}(x)$.

9



The diagram shows the curve with equation $y = x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}}$. The curve crosses the x-axis at the points A and B and has a minimum point at C.

a Find the coordinates of A and B.

b Find the coordinates of C, giving its y-coordinate in the form $a\sqrt{3} + b$, where a and b are integers.

10 $$\text{f}(x) = x^3 - 3x^2 + 4.$$

a Show that $(x + 1)$ is a factor of $\text{f}(x)$.

b Fully factorise $\text{f}(x)$.

c Hence state, with a reason, the coordinates of one of the turning points of the curve $y = \text{f}(x)$.

d Using differentiation, find the coordinates of the other turning point of the curve $y = \text{f}(x)$.