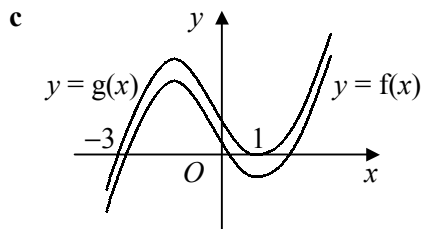**1** $= \frac{1}{3}x^3 + 4x^{\frac{3}{2}} - 3x + c$

2 **a** $f(x) = \int (1 - 6x^{-3}) \, dx$
$= x + 3x^{-2} + c$
$(1, -2) \implies -2 = 1 + 3 + c$
$c = -6$
$\therefore f(x) = x - 6 + \frac{3}{x^2}$

b $x = 2 \implies y = -\frac{13}{4}$, grad $= \frac{1}{4}$
$\therefore$ grad of normal $= -4$
$\therefore y + \frac{13}{4} = -4(x - 2)$
$4y + 13 = -16x + 32$
$16x + 4y - 19 = 0$

3 **a** $f(x) = \int (3x^2 + 2x - 5) \, dx$
$= x^3 + x^2 - 5x + c$
$(3, 22) \implies 22 = 27 + 9 - 15 + c$
$c = 1$
$\therefore f(x) = x^3 + x^2 - 5x + 1$
b $g(x) = (x + 3)(x^2 - 2x + 1)$
$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$
$= x^3 + x^2 - 5x + 3$
$= f(x) + 2$
c

4 **a** $= 2x + 6x^{-3}$
b $= \frac{1}{3}x^3 + 3x^{-1} + c$

5 grad of tangent $= 12 - 8 - 1 = 3$

tangent passes through $(0, 0)$
$\therefore$ tangent: $y = 3x$
when $x = 2$, $y = 6$

$\therefore$ curve passes through $(2, 6)$
curve: $y = \int (3x^2 - 4x - 1) \, dx$
$y = x^3 - 2x^2 - x + c$
$(2, 6) \implies 6 = 8 - 8 - 2 + c$
$c = 8$
$\therefore y = x^3 - 2x^2 - x + 8$

6 **a** $= 3\sqrt{2} - \frac{2}{\sqrt{2}}$
$= 3\sqrt{2} - \sqrt{2}$
$= 2\sqrt{2}$
b $y = \int (3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}) \, dx$
$= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$
$(4, 7) \implies 7 = 2(8) - 4(2) + c$
$7 = 16 - 8 + c$
$c = -1$
$\therefore y = 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 1$
when $x = 3$
$y = 6\sqrt{3} - 4\sqrt{3} - 1$
$y = 2\sqrt{3} - 1$

7   **a** $= \int (x^2 + 4x + 4) \, dx$

$\qquad = \frac{1}{3}x^3 + 2x^2 + 4x + c$

   b $= \int \frac{1}{4}x^{-\frac{1}{2}} \, dx$

$\qquad = \frac{1}{2}x^{\frac{1}{2}} + c$

8   **a** $f(x) = \int (3x^2 - 2x - 3) \, dx$

$\qquad = x^3 - x^2 - 3x + c$

$\qquad (-2, 0) \implies 0 = -8 - 4 + 6 + c$

$\qquad\qquad\qquad c = 6$

$\qquad \therefore \ f(x) = x^3 - x^2 - 3x + 6$

   b $x = 1 \implies y = 1 - 1 - 3 + 6 = 3$

$\qquad\qquad\qquad \text{grad} = 3 - 2 - 3 = -2$

$\qquad \therefore \ y - 3 = -2(x - 1)$

$\qquad\qquad y - 3 = -2x + 2$

$\qquad\qquad y = 5 - 2x$

9   **a** $y = \int (2x - 3x^{-2}) \, dx$

$\qquad = x^2 + 3x^{-1} + c$

$\qquad y = 0 \ \text{at} \ x = 1$

$\qquad \therefore \ 0 = 1 + 3 + c$

$\qquad\quad c = -4$

$\qquad \therefore \ y = x^2 - 4 + \dfrac{3}{x}$

   b $\dfrac{d^2y}{dx^2} = 2 + 6x^{-3}$

$\qquad \therefore \ x^2 \dfrac{d^2y}{dx^2} - 2y$

$\qquad\quad = x^2(2 + 6x^{-3}) - 2(x^2 - 4 + 3x^{-1})$

$\qquad\quad = 2x^2 + 6x^{-1} - 2x^2 + 8 - 6x^{-1}$

$\qquad\quad = 8 \qquad\qquad [\, k = 8 \,]$

10 **a** $= -\frac{1}{2}x^{-2} + c$

   b $= \int \dfrac{x^2 - 2x + 1}{x^{\frac{1}{2}}} \, dx$

$\qquad = \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \, dx$

$\qquad = \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

11 **a** $f(x) = \int (4x^3 - 8x) \, dx$

$\qquad = x^4 - 4x^2 + c$

$\qquad (2, -5) \implies -5 = 16 - 16 + c$

$\qquad\qquad\qquad c = -5$

$\qquad \therefore \ f(x) = x^4 - 4x^2 - 5$

   b $x^4 - 4x^2 - 5 = 0$

$\qquad (x^2 + 1)(x^2 - 5) = 0$

$\qquad x^2 = -1 \ [\text{no sols}] \ \text{or} \ 5$

$\qquad x = \pm\sqrt{5}$

$\qquad \therefore \ (-\sqrt{5}, 0), (\sqrt{5}, 0)$

12 **a** $y = \int (k - x^{-\frac{1}{2}}) \, dx$

$\qquad y = kx - 2x^{\frac{1}{2}} + c$

$\qquad (1, -2) \implies -2 = k - 2 + c$

$\qquad\qquad\qquad 0 = k + c \quad (1)$

$\qquad (4, 5) \implies 5 = 4k - 4 + c$

$\qquad\qquad\qquad 9 = 4k + c \quad (2)$

$\qquad (2) - (1) \quad 9 = 3k$

$\qquad\qquad\qquad k = 3$

   b $\text{grad} = 3 - 1 = 2$

$\qquad \therefore \ \text{grad of normal} = -\frac{1}{2}$

$\qquad \therefore \ y + 2 = -\frac{1}{2}(x - 1)$

$\qquad\quad 2y + 4 = -x + 1$

$\qquad\quad x + 2y + 3 = 0$