C1 SEQUENCES AND SERIES — Worksheet D

1 The second and fifth terms of an arithmetic series are 40 and 121 respectively.

a Find the first term and common difference of the series. **(4)**

b Find the sum of the first 25 terms of the series. **(2)**

2 A sequence is defined by the recurrence relation

$$u_r = u_{r-1} + 4, \quad r > 1, \quad u_1 = 3.$$

a Write down the first five terms of the sequence. **(1)**

b Evaluate $\sum_{r=1}^{20} u_r$. **(3)**

3 The first three terms of an arithmetic series are t, $(2t - 5)$ and 8.6 respectively.

a Find the value of the constant t. **(2)**

b Find the 16th term of the series. **(4)**

c Find the sum of the first 20 terms of the series. **(2)**

4 **a** State the formula for the sum of the first n natural numbers. **(1)**

b Find the sum of the natural numbers from 200 to 400 inclusive. **(3)**

c Find the value of N for which the sum of the first N natural numbers is 4950. **(3)**

5 A sequence of terms $\{u_n\}$ is defined, for $n \geq 1$, by the recurrence relation

$$u_{n+1} = k + u_n^2,$$

where k is a non-zero constant. Given that $u_1 = 1$,

a find expressions for u_2 and u_3 in terms of k. **(3)**

Given also that $u_3 = 1$,

b find the value of k, **(3)**

c state the value of u_{25} and give a reason for your answer. **(2)**

6 **a** Find the sum of the integers between 1 and 500 that are divisible by 3. **(3)**

b Evaluate $\sum_{r=3}^{20} (5r - 1)$. **(3)**

7 **a** Prove that the sum, S_n, of the first n terms of an arithmetic series with first term a and common difference d is given by

$$S_n = \tfrac{1}{2}n[2a + (n - 1)d]. \quad \textbf{(4)}$$

b An arithmetic series has first term -1 and common difference 6.

Verify by calculation that the largest value of n for which the sum of the first n terms of the series is less than 2000 is 26. **(3)**

8 A sequence is defined by the recurrence relation

$$t_{n+1} = 4 - kt_n, \quad n > 0, \quad t_1 = -2,$$

where k is a positive constant.

Given that $t_3 = 3$, show that $k = -1 + \tfrac{1}{2}\sqrt{6}$. **(6)**

9 An arithmetic series has first term 6 and common difference 3.

a Find the 20th term of the series. **(2)**

Given that the sum of the first n terms of the series is 270,

b find the value of n. **(4)**

10 A sequence of terms $t_1, t_2, t_3, \ldots$ is such that the sum of the first 30 terms is 570.

Find the sum of the first 30 terms of the sequences defined by

a $u_n = 3t_n, \quad n \geq 1,$ **(2)**

b $v_n = t_n + 2, \quad n \geq 1,$ **(2)**

c $w_n = t_n + n, \quad n \geq 1.$ **(3)**

11 Tom's parents decide to pay him an allowance each month beginning on his 12^{th} birthday. The allowance is to be £40 for each of the first three months, £42 for each of the next three months and so on, increasing by £2 per month after each three month period.

a Find the total amount that Tom will receive in allowances before his 14^{th} birthday. **(4)**

b Show that the total amount, in pounds, that Tom will receive in allowances in the n years after his 12^{th} birthday, where n is a positive integer, is given by $12n(4n + 39)$. **(4)**

12 A sequence is defined by

$$u_{n+1} = u_n - 3, \quad n \geq 1, \quad u_1 = 80.$$

Find the sum of the first 45 terms of this sequence. **(3)**

13 The third and eighth terms of an arithmetic series are 298 and 263 respectively.

a Find the common difference of the series. **(3)**

b Find the number of positive terms in the series. **(4)**

c Find the maximum value of S_n, the sum of the first n terms of the series. **(3)**

14 **a** Find and simplify an expression in terms of n for $\sum_{r=1}^{n} (6r + 4)$. **(3)**

b Hence, show that

$$\sum_{r=n+1}^{2n} (6r + 4) = n(9n + 7).$$ **(4)**

15 The nth term of a sequence, u_n, is given by

$$u_n = k^n - n.$$

Given that $u_2 + u_4 = 6$ and that k is a positive constant,

a show that $k = \sqrt{3}$, **(5)**

b show that $u_3 = 3u_1$. **(3)**

16 The first three terms of an arithmetic series are $(k + 4)$, $(4k - 2)$ and $(k^2 - 2)$ respectively, where k is a constant.

a Show that $k^2 - 7k + 6 = 0$. **(2)**

Given also that the common difference of the series is positive,

b find the 15th term of the series. **(4)**