

1 a $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$
 $a \times (\frac{3}{4})^2 = 27$
 $a = \frac{16}{9} \times 27 = 48$
 b $S_{\infty} = \frac{48}{1 - \frac{3}{4}} = 192$

2 a $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$
 $(k+4)^2 = (3k+2)(k-8)$
 $k^2 - 15k - 16 = 0$
 $(k+1)(k-16) = 0$

$$k > 0 \therefore k = 16$$

b $u_1 = 8, u_2 = 20 \therefore a = 8, r = \frac{5}{2}$
 $u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$

c $S_{10} = \frac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\,857.3$

3 a $ar = 75, ar^4 = 129.6$
 $r^3 = 129.6 \div 75 = 1.728$
 $r = \sqrt[3]{1.728} = 1.2$
 $a = 75 \div 1.2 = 62.5$
 b $u_{10} = 62.5 \times (1.2)^9 = 322.5$
 c $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 a $S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$
 $rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$
 subtracting,
 $S_n - rS_n = a - ar^n$
 $(1-r)S_n = a(1-r^n)$
 $S_n = \frac{a(1-r^n)}{1-r}$

b $\frac{2[1 - (\sqrt{2})^n]}{1 - \sqrt{2}} = 126(\sqrt{2} + 1)$
 $1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$
 $1 - (\sqrt{2})^n = 63(1 - 2)$
 $(\sqrt{2})^n = 64$
 $2^{\frac{1}{2}n} = 2^6$
 $n = 12$

5 a $\frac{18}{1-r} = 15$
 $\therefore 1-r = \frac{18}{15} = 1.2$
 $r = -0.2$
 b $u_3 = 18 \times (-0.2)^2 = 0.72$
 c $S_8 = \frac{18[1 - (-0.2)^8]}{1 - (-0.2)} = 14.9999616$
 $S_{\infty} - S_8 = 0.000\,0384$

6 a $S_3 = 5(3^3 - 1) = 130$
 $S_2 = 5(3^2 - 1) = 40$
 $u_3 = S_3 - S_2 = 90$
 b $S_{n-1} = 5(3^{n-1} - 1)$
 $u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$
 $= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$

7 a $4 \times (1.25)^7 = 19.1 \text{ mm (3sf)}$
 b GP: $a = 4, r = 1.25$
 $S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8 \text{ mm}$
 $\therefore \text{length} = 1.37 \text{ m (3sf)}$

8 a $ar = 30, ar^3 = 2.7 \therefore r^2 = 2.7 \div 30 = 0.09$
 $r > 0 \therefore r = \sqrt{0.09} = 0.3$
 $a = 30 \div 0.3 = 100$
 b $S_{\infty} = \frac{100}{1 - 0.3} = 142.9 \text{ (1dp)}$

- 9 a GP: $a = 27, r = 3$
 $S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$
- b $\sum_{r=1}^{15} 2^r$: GP, $a = 2, r = 2$
 $S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$
- $\sum_{r=1}^{15} 12r$: AP, $a = 12, d = 12$
 $S_{15} = \frac{15}{2} [24 + (14 \times 12)] = 1440$
- $\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$
- 10 a $a = 64, ar^2 - ar = 20$
 $\therefore 64r^2 - 64r = 20$
 $16r^2 - 16r - 5 = 0$
- b $(4r + 1)(4r - 5) = 0$
 $r = -\frac{1}{4}$ or $\frac{5}{4}$
- c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$
 $r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$
- d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$
- 11 a $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$
- b $u_n = 4 \times (\frac{1}{2})^{n-1}$
 $= 2^2 \times 2^{1-n}$
 $= 2^{3-n}$
- c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$
 $= 8(1 - 2^{-n})$
 $= 8 - (2^3 \times 2^{-n})$
 $= 8 - 2^{3-n}$
- 12 a $u_6 = 4 \times 3^6 = 2916$
- b GP: $a = 12, r = 3$
 $S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$
 $\therefore 6(3^t - 1) > 10^{25}$
 $3^t > \frac{10^{25}}{6} + 1$
 $t \lg 3 > \lg(\frac{10^{25}}{6} + 1)$
 $t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$
 $t > 50.8 \therefore$ smallest $t = 51$
- 13 a $a + ar^2 = a(1 + r^2) = 150$
 $ar + ar^3 = ar(1 + r^2) = -75$
 $\therefore r = -75 \div 150 = -\frac{1}{2}$
 $a = 150 \div \frac{5}{4} = 120$
- b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$
- 14 a $b - a = (3a + 4) - b$
 $2b = 4a + 4$
 $b = 2a + 2$
- b $\frac{2a + 2}{a} = \frac{6a + 1}{2a + 2}$
 $(2a + 2)^2 = a(6a + 1)$
 $2a^2 - 7a - 4 = 0$
 $(2a + 1)(a - 4) = 0$
 a integer $\therefore a = 4$
sub. $b = 10$
- 15 a after 4th bounce,
reaches $3 \times (0.6)^4 = 0.3888$ m
- b total distance
 $= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \dots]$
 $= h + 2 \times S_\infty$ of GP, $a = 0.6h, r = 0.6$
 $= h + \frac{2 \times 0.6h}{1 - 0.6}$
 $= h + 3h = 4h$ metres