1 Expand $(1 + 4x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

2 A geometric series has first term 3 and common ratio −2.

a Find the fifth term of the series. **(2)**

b Find the sum of the first ten terms of the series. **(2)**

c Show that the sum of the first eight positive terms of the series is 65 535. **(4)**

3 **a** Expand $(1 + 3x)^7$ in ascending powers of x up to and including the term in x^4, simplifying each coefficient in the expansion. **(4)**

b Use your series with a suitable value of x to estimate the value of 1.03^7 correct to 5 decimal places. **(3)**

4 Evaluate $\sum_{r=3}^{12} 2^r$. **(4)**

5 **a** Expand $(2 + x)^5$, simplifying the coefficient in each term. **(4)**

b Hence, or otherwise, write down the expansion of $(2 - x)^5$. **(1)**

c Show that

$$(2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 = k\sqrt{5},$$

where k is an integer to be found. **(4)**

6 Ginny opens a savings account and decides to pay £200 into the account at the start of each month. At the end of each month, interest of 0.5% is paid into the account.

a Find, to the nearest penny, the interest paid into the account at the end of the third month. **(4)**

b Show that the total interest paid into the account over the first 12 months is £79.45 to the nearest penny. **(5)**

7 Find the first four terms in the expansion of $(1 - 3x)^8$ in ascending powers of x, simplifying each coefficient. **(4)**

8 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ **(4)**

b Find the exact sum of the first 16 terms of the geometric series with fourth term 3 and fifth term 6. **(5)**

9 **a** Write down the first three terms in the binomial expansion of $(1 + ax)^n$, where n is a positive integer, in ascending powers of x. **(2)**

Given that the coefficient of x^2 is three times the coefficient of x,

b show that $n = \dfrac{6+a}{a}$. **(4)**

Given also that $a = \frac{2}{3}$,

c find the coefficient of x^3 in the expansion. **(3)**

10 Find the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x, simplifying each coefficient. **(4)**

11 The first term of a geometric series is 162 and the sum to infinity of the series is 486.

a Find the common ratio of the series. **(3)**

b Find the sixth term of the series. **(2)**

c Find, to 3 decimal places, the sum of the first ten terms of the series. **(4)**

12 **a** Expand $(1 + 3x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Find the coefficient of x^2 in the expansion of

$$(1 + 4x - x^2)(1 + 3x)^4.$$ **(3)**

13 In a computer game, each player must complete the tasks set at each level within a fixed amount of time in order to progress to the next level.

The time allowed for level 1 is 2 minutes and the time allowed for each of the other levels is 10% less than that allowed in the previous level.

a Find, in seconds, the time allowed for completing level 4. **(2)**

b Find, in minutes and seconds, the maximum total time allowed for completing the first 12 levels of the game. **(4)**

14 Given that

$$(1 + \frac{x}{2})^8(1 - x)^6 \equiv 1 + Ax + Bx^2 + \dots ,$$

find the values of the constants A and B. **(7)**

15 The terms of a sequence are defined by the recurrence relation

$$u_r = 2u_{r-1}, \quad r > 1, \quad u_1 = 6.$$

a Write down the first four terms of the sequence. **(1)**

b Evaluate $\sum_{r=1}^{10} u_r$. **(3)**

16 **a** Expand $(1 + x)^4$ in ascending powers of x. **(2)**

b Hence, or otherwise, write down the expansion of $(1 - x)^4$ in ascending powers of x. **(1)**

c By using your answers to parts **a** and **b**, or otherwise, solve the equation

$$(1 + x)^4 + (1 - x)^4 = 82,$$

for real values of x. **(5)**

17 The common ratio of a geometric series is 1.5 and the third term of the series is 18.

a Find the first term of the series. **(2)**

b Find the sum of the first six terms of the series. **(2)**

c Find the smallest value of k such that the kth term of the series is greater than 8000. **(4)**

18 The first two terms in the expansion of $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$, in ascending powers of x, are 2 and $90x^2$.

Given that $a < b$, find the values of the constants a and b. **(9)**