AS

Mathematics

Paper 2

Mark scheme

Specimen

Version 1.2

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	9
	Total		**1**	
2	Circles correct answer	AO1.2	B1	$y = \mathrm{f}(2x)$
	Total		**1**	
3	Correctly applies a single law of logs with either term	AO1.1a	M1	$\log_a(a^3) + \log_a\left(\frac{1}{a}\right) = 3 + (-1)$
	States correct final answer (NMS scores full marks)	AO1.1b	A1	$= 3 - 1$ $= 2$
	Total		**2**	
4	Selects an appropriate method – **either** differentiates, at least as far as: $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x....$ **or** commences completion of the square: $\left(x - \frac{5}{2}\right)^2 + ...$	AO1.1a	M1	$y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + a$ y minimised when squared bracket is 0 $\left(\frac{5}{2},\ a - \frac{25}{4}\right)$ **ALT** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 5$
	Fully differentiates **and** sets derivative equal to zero **or** fully completes square Allow one error	AO1.1a	M1	so $2x - 5 = 0$ for minimum $x = \frac{5}{2}$
	Obtains both coordinates	AO1.1b	A1	$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + a = a - \frac{25}{4}$
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Forms discriminant – condone one error in discriminant	AO1.1a	M1	for distinct real roots, disc > 0 $4^2 - 4\times3\times(2k-1) > 0$
	States that discriminant > 0 for real and distinct roots	AO2.4	R1	$16-12(2k-1) > 0$ $28-24k > 0$
	Forms an inequality from 'their' discriminant	AO1.1a	M1	$k < \frac{7}{6}$
	Solves inequality for k correctly Allow un-simplified equivalent fraction	AO1.1b	A1	
	Total		**4**	
6	States a correct integral expression (ignore limits at this stage)	AO1.1a	M1	Area = $\int_a^{2a}\left(6x^2+\frac{8}{x^2}\right)dx$
	Integrates at least one term correctly	AO1.1b	A1	$= \left[2x^3-\frac{8}{x}\right]_a^{2a}$
	Substitutes $2a$ and a into 'their' integrated expression	AO1.1a	M1	$= (16a^3-\frac{4}{a})-(2a^3-\frac{8}{a})$
	States correct final answer with terms collected FT correct substitution into incorrect integral provided both M1 marks awarded	AO1.1b	A1F	$= 14a^3+\frac{4}{a}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Divides or multiplies by $\cos\theta$	AO3.1a	M1	$\frac{\sin\theta\tan\theta}{\cos\theta}+2\frac{\sin\theta}{\cos\theta}=3$
	Obtains correct quadratic	AO1.1b	A1	$\tan^2\theta + 2\tan\theta - 3 = 0$
	Applies a correct method to solve 'their' quadratic PI	AO1.1a	M1	$(\tan\theta + 3)(\tan\theta - 1) = 0$ $\tan\theta = 1$ or -3
	Finds two correct values of tan θ from 'their' quadratic	AO1.1b	A1F	$\theta = 45°$ or $108°$ **ALT** $\sin\theta\tan\theta\cos\theta + 2\sin\theta\cos\theta = 3\cos^2\theta$
	Obtains two correct answers CAO	AO1.1b	A1	$\sin^2\theta + 2\sin\theta\cos\theta - 3\cos^2\theta = 0$ $(\sin\theta + 3\cos\theta)(\sin\theta - \cos\theta) = 0$ $\tan\theta = 1$ or -3 $\theta = 45°$ or $108°$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	Explains clearly that $\mathrm{f}(x)$ is increasing $\Leftrightarrow \mathrm{f}'(x) > 0$ (for all values of x) or Explains $\Rightarrow \mathrm{f}(x)$ is increasing $\mathrm{f}'(x) > 0$ for all values of x This may appear at any appropriate point in their argument	AO2.4	E1	For all x, $\mathrm{f}'(x) > 0 \Rightarrow \mathrm{f}(x)$ is an increasing function $\mathrm{f}(x) = x^3 - 3x^2 + 15x - 1$ $\Rightarrow \mathrm{f}'(x) = 3x^2 - 6x + 15$ $\Rightarrow \mathrm{f}'(x) = 3(x-1)^2 + 12$ $\therefore \mathrm{f}'(x)$ has a minimum value of 12 therefore $\mathrm{f}'(x) > 0$ for all values of x **OR** for $\mathrm{f}'(x)$, $b^2 - 4ac = -144$ $\therefore \mathrm{f}'(x) \neq 0$ for any real x, so $\mathrm{f}'(x)$ is either always positive or always negative. $\mathrm{f}'(0) = 15$ therefore $\mathrm{f}'(x) > 0$ for all values of x **OR** $\mathrm{f}''(x) = 6x - 6$, which = 0 when $x = 1$ so min $\mathrm{f}'(x)$ is $\mathrm{f}'(1) = 12$ therefore $\mathrm{f}'(x) > 0$ for all values of x Thus, since, $\mathrm{f}'(x) > 0$ for all values of x it is proven that $\mathrm{f}(x)$ is an increasing function.
	Differentiates – at least two correct terms	AO1.1a	M1	
	All terms correct	AO1.1b	A1	
	Attempts a correct method which could lead to $\mathrm{f}'(x) > 0$	AO3.1a	M1	
	Correctly deduces $\mathrm{f}'(x) > 0$ (for all values of x)	AO2.2a	A1	
	Writes a clear statement that links the steps in the argument together, the deduction about a positive gradient for all values of x proves that the given function is increasing for all values of x	AO2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9	States the correct gradient of the curve	AO1.2	B1	Grad of curve = $2e^{2x}$
	Forms an equation using 'their' gradient of the curve and puts it equal to $\frac{1}{2}$	AO1.1a	M1	= grad of tangent so $2e^{2x} = \frac{1}{2}$
	Takes a log of each side of 'their' equation and uses law of logs to obtain equation in x	AO1.1a	M1	$e^{2x} = \frac{1}{4} \Rightarrow 2x = \ln\left(\frac{1}{4}\right)$
	Obtains a correct exact value for x	AO1.1b	A1	$\Rightarrow x = \frac{1}{2}\ln\left(\frac{1}{4}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2$
	Substitutes 'their' value of x and obtains y value and hence the coordinates (follow through provided values are exact)	AO1.1b	A1F	$y = e^{2x} = \frac{1}{4}$ $\left(-\ln 2, \frac{1}{4}\right)$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)(i)	States correct value CAO	AO3.4	B1	50
(a)(ii)	States correct integer value CAO	AO3.4	B1	609
(b)	Forms correct equation and rearranges to obtain $e^{0.5t} = \ldots$	AO3.4	M1	$150 = 50e^{0.5t}$ so $e^{0.5t} = 3$
	Obtains the correct solution. Must give answer to 3 sf	AO1.1b	A1	$t = 2\ln 3 = 2.20$
(c)	1 mark for any clear valid reason, must be set in context of the question	AO3.5b	E1	No constraint on the number of rabbits (ie could go off to infinity) **OR** Model is only based on the 3 years of the study. Things may change **OR** Continuous model but number of rabbits is discrete **OR** Ignores extraneous factors such as disease, predation, limited food supply
(d)	Forms an equation with exponentials by letting $R = C$ PI	AO3.4	M1	$1000e^{0.1t} = 50e^{0.5t}$ $20 = e^{0.4t}$ $t = \ln 20 \div 0.4$ $= 7.49$ 2023
	Solves 'their' equation correctly	AO1.1a	M1	
	States correct answer as the year 2023 CAO NMS scores full marks for 2023	AO3.2a	A1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)(i)	States correct radius CAO	AO1.2	B1	Radius $=\sqrt{5}$
(a)(ii)	States correct centre CAO	AO1.2	B1	*C* is (7, –2)
(b)	Finds gradient of the line through the points *P* and 'their' *C* (as found in part **(a)**) Condone one sign error	AO3.1a	M1	Gradient *CP* = $\frac{-1-(-2)}{5-7} = -\frac{1}{2}$
	Correct tangent gradient obtained from 'their' *CP* gradient	AO3.1a	M1	So tangent gradient = 2
	Uses a correct form for the equation of a straight line with correct coordinates of *P* and 'their' tangent gradient	AO1.1a	M1	$y - (-1) = 2(x - 5)$
	States correct final answer in required form ($y = mx + c$) FT from 'their' *C* found in part **(a)**	AO1.1b	A1F	$y = 2x - 11$
(c)	Identifies *QTC* as a right-angled triangle PI	AO3.1a	M1	*QTC* is a right-angled triangle so we can use Pythagoras
	Finds *QC* or QC^2 FT 'their' *C* found in part **(a)**	AO1.1b	B1F	$QC^2 = (7-3)^2 + (-2-3)^2$
	Uses Pythagoras' theorem correctly for 'their' triangle	AO1.1a	M1	$4^2 + 5^2 = (\sqrt{5})^2 + QT^2$
	Correct evaluation of length of *QT* FT 'their' *QC* and 'their' radius found in part **(a)**	AO1.1b	A1F	$QT^2 = 36$ so $QT = 6$
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Begins to construct a rigorous mathematical proof by generalising the form of an even number and substituting it into the given expression	AO2.1	M1	Let $n = 2m$ $9n^2 + 6n = 9(2m)^2 + 6(2m)$ $= 36m^2 + 12m$ $= 12(3m^2 + m)$ Hence 12 is a factor of the expression $9n^2 + 6n$ when n is any even number
	Simplifies expression and extracts 12 as a common factor	AO1.1b	A1	
	Completes rigorous proof – well explained. A statement is required that links the factor of 12 to the expression $9n + 6n$ when n is an even number	AO2.1	R1	
(b)	Uses a counter example by substituting any odd number into the expression and shows that the resulting value is not a multiple of 12	AO2.2a	R1	Let $n = 1$ $9(1)^2 + 6(1) = 15$ 12 is not a factor of 15 and hence statement is not true for all integers n
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
13	Circles correct answer	AO1.1b	B1	0.26
	Total		**1**	
14	Circles correct answer	AO2.2b	B1	London
	Total		**1**	
15	Finds P(Drop and Beanstalk and Giant)	AO1.1a	M1	$\frac{80}{225}\times\frac{75}{224}\times\frac{70}{223}$
	Multiplies by 6 to obtain correct answer	AO1.1b	A1	$\frac{80}{225}\times\frac{75}{224}\times\frac{70}{223}\times 6 = 0.224$
	Total		**2**	
16(a)	Explains that Bilal may be wrong with reference to rounding	AO2.4	E1	Bilal's statement is incorrect because the figures given in the dataset are rounded to integers and therefore the actual values may total to a rounded value that is not the total of the two component rounded values.
16(b)	States that Maria's claim is not supported Explains that the actual recorded consumption values for Takeaway confectionary are non-zero with reference to knowledge of the Large Data Set… OR Just because the values happen to be 0 in these four periods values will not necessarily always be 0, with reference to knowledge of the Large Data Set	AO2.4	E1	Maria's claim is not supported because the actual data values, not the zeros that appear in the table, for the consumption of Takeaway confectionary, are not equal to zero. Maria needs to understand that when using the LDS spreadsheet the decimal values are visible (to over ten decimal places), but that the summary data shown here is rounded to the nearest integer.
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)(i)	Identifies likely outlier	AO1.2	B1	East region 2007
(a)(ii)	Finds Q_1 and Q_3 for East region FT NW if that was their outlier in part **(a)(i)**	AO1.1b	B1F	For East region $Q_1 = 0$ $Q_3 = 4$ (For NW $Q_1 = 2$ $Q_3 = 6$)
	Finds IQR for East region FT NW if that was their outlier in part **(a)(i)**	AO1.1b	B1	IQR = 4 E (IQR = 4 NW)
	Completes argument that uses formula given in the question together with 'their' values found for Q_1, Q_3 and IQR and 'their' outlier for **(a)(i)** to confirm that it is an outlier Award credit here provided 'their' values do confirm 'their' identified value to be an outlier	AO2.1	R1	$4 + 1.5\times4 = 10$ $21 > 10$ Hence East 2007 is an outlier
(b)	Explains reason for mean being unrepresentative	AO2.4	E1	Mean would be: unrepresentative as it would be affected by the large value outlier.
(c)	Provides explanation	AO2.2b	E1	Allow: Error in data entry; Some event in 2007 led to an increase in dried milk products consumption locally; Disease hit dairy herds.
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
18	Sets up enumerated population	AO3.1b	B1	Give each customer a unique number from 1 to 750.
	Explains how enumerated population will be used to obtain sample with respect to random numbers	AO2.4	E1	Generate random integers from the calculator.
	Explains how to deal with repeats	AO2.4	E1	Ignore repeats.
	Explains how to identify 50 customers	AO2.4	E1	Continue until 50 different numbers have been identified and select the corresponding customers.
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
19(a)(i)	Obtains probability from calculator	AO3.4	B1	$P(X \leq 2) = 0.678$
(a)(ii)	Obtains either of these figures (0.8791, 0.1074) PI	AO3.4	M1	$P(X \leq 3) = 0.8791$ $P(X = 0) = 0.1074$
	Obtains correct probability	AO1.1b	A1	$P(1 \leq X \leq 3) = 0.8791 - 0.1074$ $= 0.772$
(b)(i)	Recalls correct name for sampling method	AO1.2	B1	Opportunity sampling
(b)(ii)	States that sampling method is unrepresentative giving one appropriate weakness	AO3.5b	E1	The 25 students all come from the same college and cannot be said to fairly represent all students. There could be a regional difference in diet.
(b)(iii)	States both hypotheses using correct notation	AO2.5	B1	H_0: $p = 0.2$ H_1: $p > 0.2$
	States or uses B(25, 0.2) PI	AO3.3	M1	Under H_0 , use $X \sim B(25, 0.2)$ (where X represents number of students eating 5 or more portions)
	Obtains correct probability	AO1.1b	A1	$P(X \geq 8) = 0.109$
	Evaluates model by comparing $P(X \geq 8)$ with 0.05 (condone 0.0468/0.047 used instead of 0.109)	AO3.5a	M1	$0.109 > 0.05$ Hence accept H_0
	Infers H_0 accepted	AO2.2b	A1	No significant evidence that more than 20% eat at least five a day
	States correct conclusion in given context	AO3.2a	E1	
	Total		**11**	
	TOTAL		**80**	