AS Level / Year 1
Edexcel Maths / Paper 1

December 2017 Mocks

Question	Scheme		AO	Marks
1				
	$(3-k)^2-4(k)(-4)\ \{=0\}$	Uses the discriminant	AO1.1a	M1
	$\Rightarrow k^2+10k+9=0 \Rightarrow k=...$	Forms a 3TQ and attempts to solve their 3TQ	AO1.1a	dM1
	$k=-1,\ k=-9$	Correct values of *k*. Answers only scores 3/3.	AO1.1b	A1
				3
	Question 1 Notes			

1st M1 – this mark is for substituting the values of a, b and c from the quadratic into $b^2-4ac\ \{=0\}$. Looking for the correct expression only, so ignore $<,>,\leq,\geq$. Condone sign errors in substituted values of a, b and c .

2nd M1 – this mark is for forming a 3TQ and attempting to solve it using factorising, completing the square or the quadratic formula. This is dependent on the 1st M1

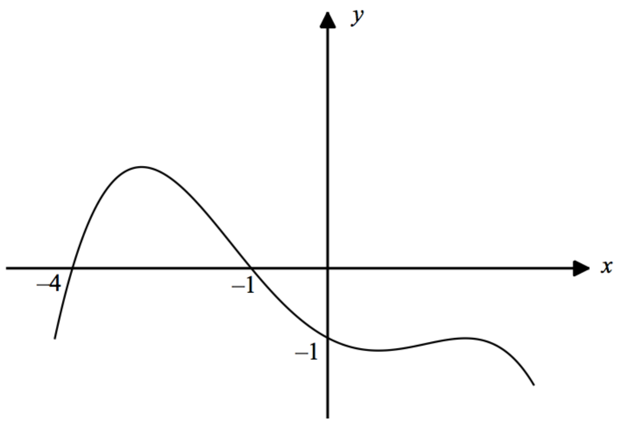
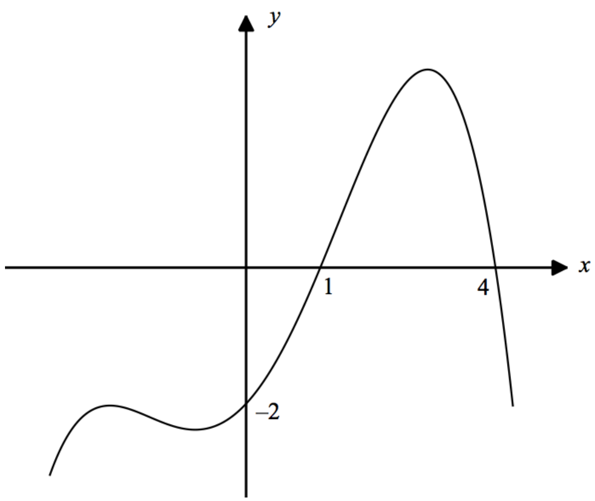
A1 – correct values of *k*, both values must be present to score the marks.

Question	Scheme		AO	Marks
2				
	$\sqrt{a}+2\sqrt{a}=3$	Uses $a^{\frac{1}{2}}=\sqrt{a}$ oe Uses $\sqrt{4a}=2\sqrt{a}$ oe	AO1.1a AO1.1a	M1 M1
	$\sqrt{a}=1\Rightarrow a=1$	Correct solution only	AO1.1b	A1
				3
	Question 2 Notes			

1st M1 – uses $a^{\frac{1}{2}}=\sqrt{a}$ OR $\sqrt{4a}=(4a)^{\frac{1}{2}}$

2nd M1 – uses $\sqrt{4a}=2\sqrt{a}$ OR $(4a)^{\frac{1}{2}}=2a^{\frac{1}{2}}$

A1 – correct solution only. Additional solutions score A0

Question	Scheme	AO	Marks
3			
(i)	 Correct shape Correct *x* intersections Correct *y* intersections	AO1.2 AO1.1b AO1.1b	B1 B1 B1 **[3]**
(ii)	 Correct shape Correct *x* intersections Correct *y* intersections	AO1.2 AO1.1b AO1.1b	B1 B1 B1 **[3]**
			6

	Question 3 Notes

(i) and (ii):

1st B1 – correct shape, with the turning points of the graph in the correct quadrants and the orientation of the curve correct

2nd and 3rd B1 – intersection points must be clearly labelled (and correct). Accept values on the axes for this or coordinates. Do **not** penalise candidates that have the coordinates the wrong way, e.g. (0, –4) instead of (–4, 0), <u>if it is clearly positioned on the correct coordinate axis and by the correct point.</u>

Question	Scheme		AO	Marks
4				
(a)	$12 = 6\lambda \Rightarrow \lambda = 2$	Finds correct ratio of two vectors oe	AO3.1a	M1
	$-a = \lambda(9-5a) \Rightarrow -a = 2(9-5a) \Rightarrow a = ...$	Uses their ratio to find *a*	AO1.1a	M1
	$a = 2$	Correct value of *a* only	AO1.1b	A1 **[3]**
(b)	$\mathbf{r} = -6\mathbf{i} + \mathbf{j}$	Correct **r** ft their value of *a* in (a)	AO1.1b	B1FT
	$\lvert\mathbf{r}\rvert = \sqrt{(-6)^2 + 1^2} = \sqrt{37} \quad (6.08276...)$	Uses length of a vector formula Correct length of **r** oe	AO1.1a AO1.1b	M1 A1 **[3]**
				6

Question	Scheme		AO	Marks
5				
(a)	$\int_0^4 \left(px - 18\sqrt{x}\right)dx = -48$	Sets up correct equation with correct limits and –16. This can be implied and seen at any point (see notes)	AO3.1a	B1
	$\left.\frac{px^2}{2} - 12x^{\frac{3}{2}}\right\|_0^4 \quad \{= -48\}$	Attempts to integrate indefinitely	AO1.1a	M1
		Correct indefinite integration (ignore constants of integration)	AO1.1b	A1
	$\frac{p(4)^2}{2} - 12(4)^{\frac{3}{2}} = -48 \Rightarrow p = ...$	Substitutes limits in the correct way around **and** attempts to solve their equation (RHS can be any value)	AO1.1a	dM1
	$p = 6$	Correct p	AO2.1	A1 **[5]**
(b)	$f'(x) = 6 - \frac{9}{\sqrt{x}}$	Differentiates $f'(x)$ w.r.t x with some value for p	AO1.1a	M1
	$6 - \frac{9}{\sqrt{x}} = 0 \left\{\Rightarrow x = \frac{9}{4}\right\}$	Sets their derivative = 0 and attempts to solve for x	AO1.1a	dM1
	$f(1) = 6\left(\frac{9}{4}\right) - 18\sqrt{\frac{9}{4}} \left\{= -\frac{27}{2}\right\}$	Substitutes their x into f	AO1.1a	dM1
	So coordinates of min. point are $\left(\frac{9}{4}, -\frac{27}{2}\right)$	Correct coordinates	AO1.1b	A1 **[4]**

(c)	$f''(x)=\frac{9}{2\sqrt{x^3}}$	Differentiates their $f'(x)$ again, can be implied	AO1.1b	M1
	$f''\left(\frac{9}{4}\right)=\frac{3}{\sqrt{(9/4)^3}}\left\{=\frac{8}{9}\right\}>0$	Substitutes 1 into the second derivative **and** shows it is positive. Note that this requires the second derivative to be **correct**	AO2.1	A1
	Since $f''(1)>0$, $\left(\frac{9}{4},-\frac{9}{2}\right)$ is a minimum point	Conclusion	AO2.4	A1 **[3]**
				12

Question 5 Notes

(a)

B1 – this mark can be awarded at any stage. It can either be awarded for the equation written explicitly or implied through the workings.

For the implication, we need to see evidence that the candidate has evaluated a definite integral corresponding to $\int_0^4 \left(px-18\sqrt{x}\right)dx$ (the actual computation need not be correct) and set this equal to <u>**– 48**</u>. NB: $\int_4^0 \left(px-18\sqrt{x}\right)dx=48$ is equivalent and scores M1.

2nd M1 – for an attempt to integrate at least **one** of the terms in $px-18\sqrt{x}$ indefinitely wrt *x* (add one to the power, divide by the new power). Need not see 48/–48 anywhere.

1st A1 – correct indefinite integration of $px-18\sqrt{x}$ wrt *x*.

3rd M1 – substitutes their limits in **AND** attempts to solve the equation for *p*. Note that any value can appear on the RHS and any limits are OK, as long we see an equation. Some candidates might evaluate the definite integral and *then* equate this value to a number, this is OK and marks can be awarded when you see the equation.

(c) 1st A1 – this is an accuracy mark and is not FT. They require the correct *x* coordinate of the min. point from (b) and the correct second derivative. They don't need to evaluate their second derivative at 9/4; substituting it in, followed by a > sign is enough to 'show' it is positive as it is trivial.

2nd A1 – conclusion: 'states that the second derivative at 9/4 is positive' and '**so** (9/4, –27/2)' is a minimum. Accept 'it' for (9/4, –27/2).

Question	Scheme		AO	Marks
6				
(a)	$\frac{BD}{\sin 50} = \frac{10}{\sin 77}$	Forms correct equation oe	AO1.1a	M1
	$\therefore AD = BD = 7.86(1945...)$ {cm}	Correct length of *AD*. Awrt 7.86	AO1.1b	A1 **[2]**
(b)	Area of entire sector $= \frac{\pi r^2}{4} = \frac{\pi(7.8619...)^2}{4} = 48.545...$ {cm^2}	Correct area of sector ft their *AD*	AO2.2	B1F
	Area of triangle *ABD* $= \frac{1}{2}(7.8619...)^2 = 30.9050...$ {cm^2}	Attempts to find area of *ABD*	AO1.1a	M1
	Area of *R* $= 48.545... - 30.905... = 17.6\{4051...\}$ {cm^2}	Correct area. Awrt 17.6	AO1.1b	A1 **[3]**
(c)	$DC = \frac{10\sin 53}{\sin 77} = 8.19642...$ {cm}	Attempts to find length of *DC*	AO1.1a	M1
	$AC = \frac{2\pi r}{4} = \frac{2\pi(7.8619...)}{4} = 12.34951...$	Attempts to find length of arc *AC*	AO2.2	M1
	Perimeter = $10 + 8.19642... + 7.86194... + 12.34951...$ = 38.4 {cm}	Adds all the relevant lengths together Correct perimeter to 1 dp, cao	AO1.1a AO1.1b	dM1 A1 **[4]**
				9

	Question 6 Notes
(a) M1 – **attempts** to find the length of *AD* or *BC.* The sine rule must be used correctly, i.e. the angles/lengths used must be consistent. A1 – correct length of *AD*. **(c)** 1st M1 – **attempts** to find *DC*. There are a variety of approaches here. Can use the sine or cosine rule. The general principle is: the formula used must be correct and the correct values must be substituted in to score the M1. A1 – correct perimeter to 1 dp. Cao	

Question	Scheme		AO	Marks
7				
(a)	$\left(2-\frac{1}{\sqrt{x}}\right)^8 = 2^8 + \binom{8}{1}(2)^7\left(-\frac{1}{\sqrt{x}}\right)^1 + \binom{8}{2}(2)^6\left(-\frac{1}{\sqrt{x}}\right)^2 + \binom{8}{3}(2)^5\left(-\frac{1}{\sqrt{x}}\right)^3 + \ldots$	See notes for mark breakdown	AO1.1b AO1.1a AO1.1b AO1.1b	B1 M1 A1 A1
	$\left(2-\frac{1}{\sqrt{x}}\right)^8 = 256 - \frac{1024}{\sqrt{x}} + \frac{1792}{x} - \frac{1792}{\sqrt{x^3}} + \ldots$	Correct binomial expansion oe (accept $\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$ etc.)	AO1.1b	A1 oe **[5]**
(b/i)	$(1+x)^r = 1^r x^0 + \binom{r}{1}x^1 + \binom{r}{2}x^2 + \ldots + 1^0 x^r$ $= 1 + rx + \binom{r}{2}x^2 + \ldots + x^r$	Uses the binomial expansion. Need to see 1 and rx appearing clearly. Some candidates may also use $(1+x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \ldots$	AO2.1	M1
	$\binom{r}{2}x^2 + \ldots + x^r \geq 0$, since $x > 0$, so $(1+x)^r \geq 1 + rx$	Convincing proof with an explanation. Accept $>$.	AO2.4	A1 **[2]**
(b/ii)	LHS = $(1+0)^r = 1^r = 1$ RHS = $1 + r(0) = 1$ LHS = RHS, so true for $x = 0$	Correct verification	AO2.1	B1 **[1]**

(b/iii)	e.g. let $x=-3$, then if $r=5$, we have $\text{LHS} = (1-3)^5 = -32$ $\text{RHS} = 1+5(-3) = -14$	Attempts to use a suitable counter-example: picks a value of $x < -1$, substitutes it into the LHS and RHS with a fixed r and attempts to LHS < RHS	AO2.1	M1
	LHS < RHS, therefore it is not true for $x < -1$.	Convincing proof with conclusion	AO2.1	A1 **[2]**
				10

Question 7 Notes

(a) B1 – sight of 2^8 as the constant term oe

1st M1 – term of the form $\binom{8}{r}(2)^{8-r}\left(-\frac{1}{\sqrt{x}}\right)^r$ or equivalent for any r, $r \neq 0,8$ (in particular, accept $8-r$ and r switched).

1st A1 – at least **any** two terms of the expansion correct, unsimplified or better

2nd A1 – the four terms required terms given, unsimplified or better. Ignore extra terms

3rd A1 – correct expansion, with each term simplified. Accept equivalent simplified forms. Ignore extra terms

SC: first 4 terms in ascending powers of *x* (or any other 4 terms given instead of the first 4 descending in *x*) can score at most B1M1A1A0A0.

(b/i) M1 – uses the binomial expansion with the terms 1 and *rx* clearly appearing. Needs to show one extra term **and** the final term. '+ …' without a final term is not sufficient as this can suggest the series is infinite.

A1 – justifies the inequality by stating that the other terms are positive.

Note: Accept $>$ instead of $\geq$.

(b/iii) M1 – a suitable counter-example: for this mark, candidates need to pick a value of $x < -1$ and attempt to show that this value does not satisfy the inequality for any **integer** *r* (i.e. substituting into both sides of the inequality). NOTE: *r* does not have to be non-negative or arbitrary.

A1 – complete and convincing proof with a conclusion.

Question	Scheme		AO	Marks
8				
(i)	$5 = a(4^b)$, $12 = a(8^b)$	Forms the two **correct** equations in *a* and *b*	AO3.1a	M1
	$\Rightarrow \frac{5}{12} = \frac{4^b}{8^b} \Rightarrow 2^b = \frac{12}{5}$	Eliminates one of the variables from their equations and attempts to reduce it to a log problem	AO1.1a	M1
	$b = \frac{\log\left(\frac{12}{5}\right)}{\log 2} = 1.263...$	Takes logs (dependent on 2nd M1 **only**)	AO1.1a	dM1
	$\Rightarrow a = \frac{5}{4^{1.263...}} = 0.8680....$	Correct values of *a* and *b*, awrt 0.868 and 1.26	AO1.1b	A1 **[4]**
(ii/a)	$y = pn^q$ for some p and q , where q is the gradient of the line and $\ln p$ is the y-intercept	Seen or implied through workings	AO2.2	M1
	$m = \frac{2.79 - -22.77}{5 - 45}$ $\{= -0.639\}$	Attempts to find the gradient of the line	AO3.1b	M1
	$c = 2.79 - ('-0.639')(5)$ $\{= 5.985\}$	Attempts to find the *y* intercept of the line. Dependent on 2nd M1 **only**	AO3.1b	dM1
	$\Rightarrow q = -0.639,\ p = e^{5.985} = 397.422...$			
	$\therefore y \approx 397n^{-0.639}$	Expresses *y* in terms of *n*. Awrt $y \approx 397n^{-0.639}$. Don't need $\approx$ (accept =)	AO3.3	A1 **[4]**

(ii/b)	$y = 397.422(87)^{-0.639} = 22.9\ (22.90335...)$	Substitutes 87 into **their** (i/a) Awrt 22.9	AO3.4 AO3.4	M1 A1 **[2]**
(ii/c)	Unreliable, because 87 is outside of the data range / the model may not be suitable for atomic numbers larger than 55 / oe	Unreliable + reason (see notes)	AO3.5b	B1 **[1]**
				11

Question 8 Notes

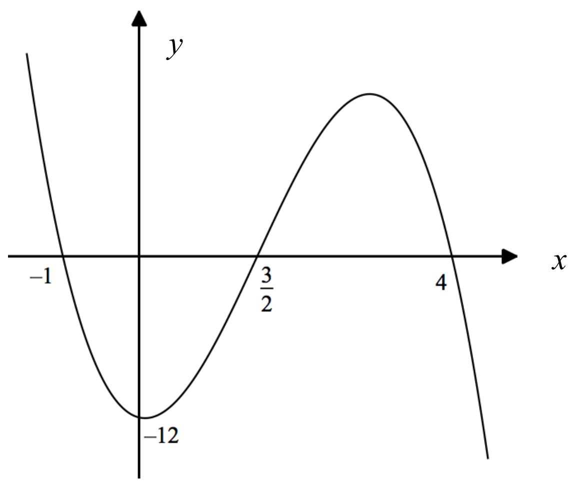
(i) 2nd M1 – eliminates one of the variables from **their** equations. Their equations **must** involve variable powers for this mark.

3rd M1 – takes logs to solve their equation in one variable. This is dependent on the 2nd M1 (Note: it is not dependent on the 1st M1)

(ii/a) 1st M1 – this can be implied at any stage of the workings. Candidates need to show they recognise that *y* and *n* are connected by a power relation and that they know **how** to work out *y* and *n*. This can be implied from the correct answers or candidates choosing their gradient as the power and $e^{\text{their intercept}}$ as '*p*'.

(ii/c) B1 – states that it is unreliable and gives a reason. Reason needs to state that the 87 is outside of the data range (owtte), or that the model may not be suitable for atomic numbers larger than 55, or equivalent. They must make it clear that 87 is not within the data range, even if not explicitly stated. Accept 'this is extrapolation' oe.

Question	Scheme		AO	Marks
9				
(a)	$\mathrm{f}(4) = -2(4)^3 + 9(4)^2 - (4) - 12$ $= -128 + 144 - 4 - 12$ $= 0$ $\therefore$ the curve y = f(x) crosses the *x* axis when *x* = 4	Substitutes 4 into f **and shows** that it is equal to 0 (all of the terms or groups of terms must be evaluated) **and** concludes: 'therefore, it crosses {the *x* axis} when *x* = 4' oe	AO2.1	B1 **[1]**
(b) **Method 1**	$\begin{array}{r} -2x^2 + x + 3 \\ x-4 \overline{)\,-2x^3 + 9x^2 - x - 12} \\ \underline{-2x^3 + 8x^2} \\ +1x^2 - 1x \quad (*) \\ \underline{+1x^2 - 4x} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$	Attempts to find the other quadratic factor by long division All working leading up to and including the line (*) correct, including the $-2x^2$ in the quotient	AO1.1a AO1.1b	M1 A1
	$\therefore \mathrm{f}(x) = (x-4)(-2x^2 + x + 3)$ $= -(x-4)(2x-3)(x+1)$	Attempts to factorise **their** quadratic factor Correct factorisation oe (e.g. accept –ve sign distributed into one of the factors)	AO1.1a AO1.1b	dM1 A1 **[4]**

(b) **Method 2**	$f(x)=(x-4)(-2x^2+bx+3)$	Attempts to find out quadratic factor using inspection Quadratic factor of the form $-2x^2+bx+3$	AO1.1a AO1.1b	M1 A1
	$\therefore f(x)=(x-4)(-2x^2+x+3)$ $=-(x-4)(2x-3)(x+1)$	Attempts to factorise **their** quadratic factor Correct factorisation oe (e.g. accept –ve sign distributed into one of the factors)	AO1.1a AO1.1b	dM1 A1 **[4]**
(c)		Correct shape (see notes) Correct *x* intersections Correct *y* intersection	AO1.2 AO1.1b AO1.1b	B1 B1 B1 **[3]**
(d)	$x-4=-1,\ \frac{3}{2},\ 4$ $\Rightarrow x=3,\ \frac{11}{2},\ 8$	Sets $x-4$ equal to at least one of the roots of their graph in (c) Correct values of *x*	AO2.2 AO1.1b	M1 A1 **[2]**
				10

	Question 9 Notes

(a) B1 – substitutes 4 into f, **shows** that it is 0 and then gives a conclusion. At least some of the terms have to be evaluated here, either individually or in groups. For example,

$f(4) = -2(4)^3 + 9(4)^2 - (4) - 12 = 0$ is NOT enough and scores B0, but $f(4) = -2(4)^3 + 9(4)^2 - (4) - 12 = 16 - 16 = 0$ is OK

The conclusion requires something simple: 'therefore, it crosses the *x* axis at 4' oe

(b) Method 1:

2nd M1 – writes their factor $ax^2 + bx + c = (px + r)(qx + s)$, where $pq = \pm a,\ rs = \pm c$ (M0 if their quadratic factor is irreducible)

(b) Method 2:

1st M1 – attempts to find the quadratic factor by inspection. This mark is awarded for one the coefficients of the quadratic factor correct.

1st A1 – their quadratic factor of the form $-2x^2 + bx + 3$ for some number **or letter** *b*.

2nd M1 – writes their factor $ax^2 + bx + c = (px + r)(qx + s)$, where $pq = \pm a,\ rs = \pm c$ (M0 if their quadratic factor is irreducible)

(c)

1st B1 – correct shape of a cubic, with two turning points and the correct orientation. **Condone** if the left-most is not in the +ve *x* quadrant, but anywhere between $-1 < x < 3/2$.

2nd and 3rd B1 – accept values plotted directly on the *x* axis or coordinates. Do **not** condone coordinate confusion in this question.

Question	Scheme		AO	Marks
10				
	$\frac{dy}{dx}=\frac{16x^2-9}{(3-4x)}=-4x-3$	Attempts to simplify dy/dx by factorising out and cancelling *x* Correct simplified dy/dx	AO3.1a AO1.1b	M1 A1
	$\Rightarrow y=\int(-4x-3)\,dx=-2x^2-3x+k$	Integrate their dy/dx indefinitely and correctly. **Use of definite integration is SC (see notes)**	AO3.1a	dM1
	$-12=-2(2)^2-3(2)+k\Rightarrow k=2$	Subs (2, –12) into their g to find *k* Correct value of *k*	AO1.1a AO1.1b	dM1 A1
	$\therefore \mathrm{g}(x)=-2x^2-3x+2$			
	$\mathrm{g}(x)=-2\left\{\left(x-\frac{3}{4}\right)^2-\frac{9}{16}\right\}+2$ $\Rightarrow \mathrm{g}(x)=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}$ so $a=-2$, $b=\frac{3}{4}$ and $c=\frac{25}{8}$	Attempts to complete the square. See notes for details Correct values of a, b and c stated	AO1.1a AO2.1	dM1 A1
				7

	Question 10 Notes

1st M1 – cancels *x* from top and bottom

1st A1 – completely correct simplified dy/dx

2nd M1 – integrates **their** simplified dy/dx indefinitely and correctly. + constant is necessary. Accept any letter, including *c* – *k* is used in the scheme to avoid confusion with *c* later. This is dependent on the 1st M1.

3rd M1 – substitutes the point (2, –12) into their g correctly and attempts to find their constant. This is dependent on previous M marks.

2nd A1 – correct constant of integration.

4th M1 – attempts to complete the square on their g. We need to see them extract their leading coefficient, and then use $x^2 \pm 2b = (x \pm b)^2 - b^2$ for the remaining factor. If they extract the leading coefficient from the whole expression, ignore what they do with the constant.

3rd A1 – correct values of a, b and c stated. Note that g(x) alone is not sufficient here (it is not what the question asked for).

Special case 1: $\frac{dy}{dx} = -4x - 3 \Rightarrow \int_{-12}^{y} dy' = \int_{2}^{x} (-4x' - 3) dx'$ scores 2nd M1 (no need for primes). Integrating and substituting the limits on both sides scores 3rd M1. The correct expression for g gives the 2nd A1. Then the rest is as per the original scheme.

Alternative:

Candidates may find g' and plug this into the expression for dy/dx and compare coefficients. This is unlikely to be seen and should be sent to review if seen.

Question	Scheme		AO	Marks
11				
(a)	$qx = -2y + 4 \Rightarrow y = ...$ $\left\{ y = -\frac{q}{2} + 2 \right\}$	Attempts to make y the subject	AO1.1a	M1
	Gradient of $l = \frac{2}{q}$	Correct gradient of l	AO1.1a	A1 **[2]**
(b)	$\frac{dy}{dx} = -2x^{-3} + \frac{3}{2p}x^{-\frac{1}{2}}$	Attempts to find dy/dx	AO3.1	M1
	$\left.\frac{dy}{dx}\right\|_{x=1} = -2(1)^{-3} + \frac{3}{2p}(1)^{-\frac{1}{2}} = -2 + \frac{3}{2p}$	Substitutes 1 into **their** dy/dx Correct value of dy/dx at 1	AO1.1a AO1.1b	dM1 A1
	$\therefore -2 + \frac{3}{2p} = \frac{2}{q} \Rightarrow p = ...$	Sets **their** dy/dx at 1 equal to **their** (a) and attempts to re-arrange for p	AO3.1	dM1
	$p = \frac{3q}{4q+4}$	Correct expression of p in terms of q	AO1.1b	A1 **[5]**
				7

Question 11 Notes

(b) 1st M1 – attempts to differentiate **both** terms

3rd M1 – this is for setting the value of their gradient of the **tangent** to C at 1 ($\left.\frac{dy}{dx}\right|_{x=1}$) = their (a). If they use the gradient of the normal, it is M0.

Question	Scheme		AO	Marks
12				
(a)	$\theta = \cos^{-1}(-0.3) = 107.457...$	Finds principal value of θ	AO1.1a	M1
	$\theta = 180 + 72.5423...,\ -180 + 72.5423...$	Attempts to find one other value of θ in range (see notes)	AO1.1a	dM1
	$\theta =$ {awrt} $-107°,\ 107°,\ 253°$	Correct values of θ	AO1.1b	A1 **[3]**
(b/i)	$(a-b)(a^2+ab+b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$ $= a^3 - b^3$	Expands brackets and gives a convincing proof	AO2.1	B1 **[1]**
(b/ii)	$\dfrac{1+\sin x\cos x}{\cos^3 x - \sin^3 x} \equiv \dfrac{1+\sin x\cos x}{(\cos x - \sin x)(\cos^2 x + \sin x\cos x + \sin^2 x)}$ $\equiv \dfrac{1}{\cos x - \sin x}$	Factorises $\cos^3 x - \sin^3 x$ using (b/i)	AO3.1a	M1
	$\therefore \dfrac{1+\sin x\cos x}{\cos^3 x - \sin^3 x} + \dfrac{1}{\sin x + \cos x} \equiv \dfrac{1}{\cos x - \sin x} + \dfrac{1}{\sin x + \cos x}$ $\equiv \dfrac{\cos x - \sin x + \sin x + \cos x}{\cos^2 x - \sin^2 x}$ $\equiv \dfrac{2\cos x}{\cos^2 x - \sin^2 x}$	Uses a common denominator to combine first and second terms Convincing proof	AO1.1a AO2.1	dM1 A1 **[3]**

(b/iii)	$\text{LHS} \equiv \frac{2\cos x}{\cos^2 x - \sin^2 x} + \frac{\sin^2 x - 2\cos x - 1}{\cos^2 x - \sin^2 x}$ $\equiv \frac{\sin^2 x - 1}{\cos^2 x - \sin^2 x}$ $\equiv -\frac{\cos^2 x}{\cos^2 x - \sin^2 x}$ $\equiv -\frac{1}{1 - \tan^2 x}$ $\equiv \frac{1}{\tan^2 x - 1}$	Combines the two fractions and uses $\sin^2 x - 1 \equiv -\cos^2 x$ Divides by $\cos^2 x$ and uses $\tan x = \frac{\sin x}{\cos x}$ Convincing proof with no errors seen	AO2.1 AO2.1 AO2.1	M1 dM1 A1 **[3]**
				10

Question 12 Notes

(a) 2nd M1 – attempts to find the other values of θ in range. This is dependent on the 1st M1. If they have the principal value wrong, this value must be obtuse in order for them to get the 2nd M1 for the correct method to find other values.

(b/ii) 1st M1 – uses the identity in (b/i) to factorise $\cos^3 x - \sin^3 x$. They don't need to do anything with the factorisation for this mark, this mark is just for the correct factorisation oe. We do **need** to see the whole factorisation before candidates jump to $1 + \sin x \cos x$, as this is a proof question.

Question	Scheme		AO	Marks
13				
(a) Method 1	$\lim_{h\to 0}\left(\frac{(x+h)^3-x^3}{h}\right)=\lim_{h\to 0}\left(\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}\right)$ $=\lim_{h\to 0}\left(3x^2+3xh+h^2\right)$ $=3x^2$	1st M1: considers $\frac{(x+h)^3-x^3}{h}$ 2nd M1: attempts to simplify numerator A1: convincing proof, clearly considering the limit	AO2.1 AO1.1a AO2.1	M1 dM1 A1 **[3]**
(a) Method 2	$\lim_{x\to c}\left(\frac{x^3-c^3}{x-c}\right)=\lim_{x\to c}\left(\frac{(x-c)(x^2+xc+c^2)}{(x-c)}\right)$ $=\lim_{x\to c}\left(x^2+xc+c^2\right)$ $=3c^2$ $\Rightarrow (x^3)'=3x^2$	1st M1: considers $\frac{f(x)-f(c)}{x-c}$ 2nd M1: simplifies to x^2+xc+c^2 A1: convincing proof, clearly considering the limit, giving final answer in terms of *x*	AO2.1 AO1.1a AO2.1	M1 dM1 A1 **[3]**
(b)	$\lim_{h\to 0}\frac{(x+h)^3-x^3}{\sqrt{x+h}-\sqrt{x}}=\lim_{h\to 0}\frac{\frac{(x+h)^3-x^3}{h}}{\frac{\sqrt{x+h}-\sqrt{x}}{h}}$ $=\frac{\frac{d}{dx}(x^3)}{\frac{d}{dx}(\sqrt{x})}=\frac{3x^2}{0.5x^{-1/2}}=6x^{\frac{5}{2}}$	Recognises that the limit is the ratio of two derivatives, identifying **either** x^3 or $\sqrt{x}$ as being the functions being differentiated Identifies that the limit is $\frac{\frac{d}{dx}(x^3)}{\frac{d}{dx}(\sqrt{x})}$ Correct limit oe	AO2.2 AO2.2 AO2.1	M1 A1 A1 **[3]**
				6

	Question 13 Notes
(b) 1st M1 – clearly identifies that the limit is the ratio of two derivatives and identifies correctly one of the two functions 1st A1 – reduces limit to $\dfrac{\frac{d}{dx}\left(x^3\right)}{\frac{d}{dx}\left(\sqrt{x}\right)}$ 2nd A1 – correct limit = $6x^{\frac{5}{2}}$ oe	

Marks breakdown by AO

AO	Number of marks	%
AO1	62	62
AO2	24	24
AO3	14	14