

Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				
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Examiner Comments	

Total Marks

# PAPER 1

## ADVANCED SUBSIDIARY

# CM

Practice Paper B

Time allowed: 2 hours

### Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

### Information to candidates:

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 12 questions in this question paper. The total mark for this paper is 100.

### Advice to candidates:

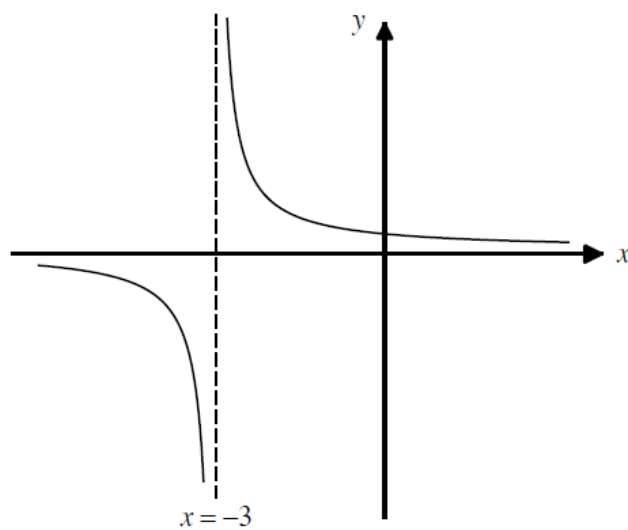
- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

AS/B1

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1 **Figure 1** shows a sketch of the curve  $C$  with equation  $y = \frac{1}{x+k}$ , where  $k$  is a constant.



**Figure 1**

(a) Write down the value of  $k$ . (1)

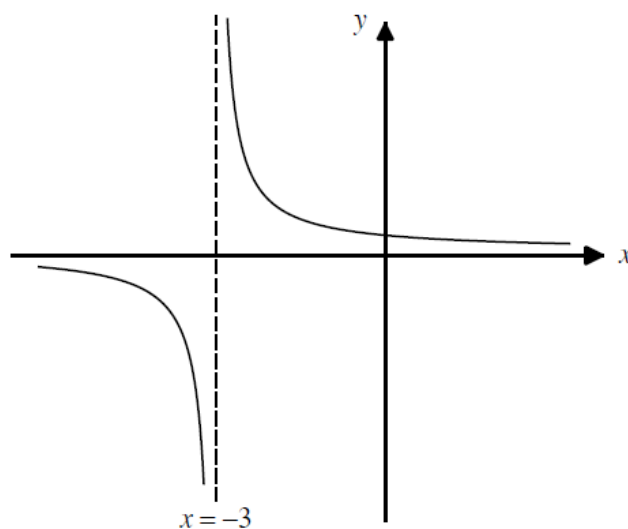
The sketch of the curve  $C$  in **Figure 1** is given on page 3.

(b) On these axes, sketch the curve with equation  $y = x^3 - 4x^2 + 4x$ .

On your sketch, you should show clearly the coordinates of any points where the curve crosses or meets the coordinate axes. (3)

(c) State, with a reason, the number of solutions to the equation

$$x^3 - \frac{1}{x+k} = 4x^2 - 4x \quad (2)$$



2 Evaluate

$$\int_1^4 (\sqrt{x} - 2x^{-3} + 4) dx$$

giving your answer as an exact value in its simplest form. (5)

3 Three forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  act on a particle  $P$ . Given that

$$\mathbf{F}_1 = (4\mathbf{i} + 6\mathbf{j}) \text{ newtons}$$

$$\mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j}) \text{ newtons}$$

$$\mathbf{F}_3 = (-5\mathbf{i} - 7\mathbf{j}) \text{ newtons}$$

(a) find the magnitude of the resultant force  $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$  acting on  $P$ . (3)

A fourth force  $\mathbf{F}_4$  newtons now also acts on  $P$ . The new resultant force on  $P$  is  $\mathbf{F}_{R^*}$ , where

$$\mathbf{F}_{R^*} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4. \text{ Given that } \mathbf{F}_{R^*} \text{ acts at a bearing of } 045^\circ,$$

(b) find, in terms of  $\mathbf{i}$  and  $\mathbf{j}$ , possible components of the force  $\mathbf{F}_4$ . (3)

4 (a) Show in clear stages that

$$\frac{a + \sqrt{b}}{c + \sqrt{d}} = \frac{ac}{c^2 - d} + \frac{-a\sqrt{d} + c\sqrt{b} - \sqrt{bd}}{c^2 - d}$$

where  $a, b, c$  and  $d$  are positive constants. (5)

Given that  $m$  is rational,

(b) find the value of  $m$  such that  $\frac{m + \sqrt{2}}{1 + \sqrt{8}}$  is rational. (2)

5 (a) Differentiate  $6x^2 + 1$  from first principles with respect to  $x$ . (4)

The curve  $C$  has the equation  $y = f(x)$ . Given that

$$f(x) = x^3 - 2x^2 + 11x - 4$$

(b) show that there is no normal line to  $C$  with gradient  $-\frac{1}{2}$ . (7)

- 6 The number of plants infected with a disease in a field varies according to the formula

$$N = 90(2 - e^{-0.05t})$$

where  $N$  is the number of plants infected with the disease and  $t$  is the time, in days, since the outbreak. There are 150 plants in the field at risk of being infected with the disease.

- (a) Find the number of plants infected with the disease at the time of outbreak. (2)
- (b) Calculate  $\frac{dN}{dt}$ . (2)
- (c) Use your answer to part (b) to explain why the number of plants infected with the disease is increasing with time. (1)
- (d) (i) Explain why the model will eventually become unrealistic. (1)
- (ii) Calculate the value of  $t$  at which the model becomes unrealistic. (3)

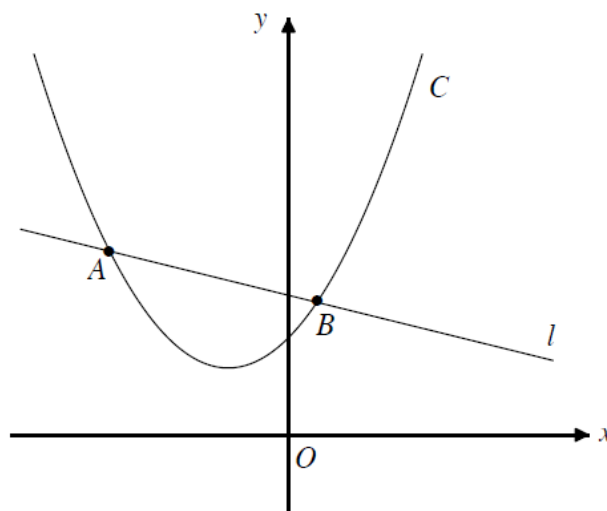
- 7 The straight line  $l$  passes through the points  $(4,5)$  and  $(12,-3)$ .

- (a) Find the equation of the straight line  $l$  giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are constants to be found. (4)

The curve  $C$  has the equation  $y = 3x^2 + 4x + 7$ .

The curve  $C$  and the straight line  $l$  intersect at the points  $A$  and  $B$ .

**Figure 2** below shows the curve  $C$ , the straight line  $l$  and the points  $A$  and  $B$ .



**Figure 2**

- (b) Find the coordinates of the points  $A$  and  $B$ . (5)

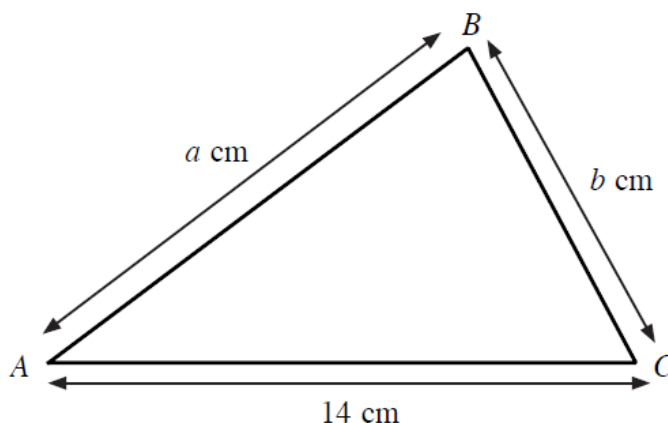
The line  $l$  crosses the  $y$  axis at the point  $P$ .

- (c) Show that the ratio  $|AP| : |BP|$  is  $m : 1$ , where  $m$  is an integer to be found. (3)

- 8 The coefficient of the term in  $x^6$  in the expansion of  $(1 - px)^{15}$  is 3648645.

- (a) Find the value of the constant  $p$ . (3)
- (b) Find the coefficient of the term in  $x^8$ , giving your answer to four significant figures. (2)

- 9 The triangle  $ABC$  has  $AB = a$  cm,  $BC = b$  cm and  $AC = 14$  cm, as shown in **Figure 3**. The perimeter of the triangle  $ABC$  is 40 cm.



**Figure 3**

Given that the angle  $ACB = \theta$ ,

(a) show that  $\cos \theta = \frac{13}{7} - \frac{120}{7b}$ . (4)

(b) Show that the area of the triangle  $A$  cm<sup>2</sup> satisfies  $A^2 = -120b^2 + 3120b - 14400$ . (4)

(c) (i) Find the maximum area of the triangle  $ABC$ . (4)

(ii) State the type of triangle  $ABC$  is when its area is a maximum. (1)

10 (a) Use a counterexample to show that if  $n$  is an integer,  $n^2 + 1$  is not necessarily prime. (2)

(b) By manipulating the indices, prove that  $(-1)^{x+y} = (-1)^{x-y}$  for integers  $x$  and  $y$ . (3)

(c) Explain how part (b) can be used to prove that  $x + y$  is even if and only if  $x - y$  is even. (1)

- 11 (a) Prove that

$$\frac{1}{\sin x} - \sin x \equiv \frac{\cos x}{\tan x}$$

(2)

(b) Hence, or otherwise, solve the equation

$$4 + \sin^2 X (\cos^2 X)^{-1} = \frac{2}{\cos X} \left( \sin X - \frac{1}{\sin X} \right) + \tan^2 X$$

for  $-360^\circ \leq X \leq 360^\circ$ .

(6)

**12** (a) Show that the  $x = 1$  is the only real root of the equation  $4x^3 + 6x - 10 = 0$ . **(5)**

The variable point  $P(x, y)$  lies on the curve  $C$  with equation  $y = x^2$ . The point  $Q$  has the coordinates  $(5, -1)$ .

(b) Show that the distance  $d$  between the points  $P$  and  $Q$  satisfies the equation

$$d^2 = x^4 + 3x^2 - 10x + 26$$
**(3)**

(c) Hence, or otherwise, use calculus to find the coordinates of the point on  $C$  that is closest to the point  $Q$ . Justify that your point on  $C$  is the closest point to  $Q$  by further calculus. **(5)**