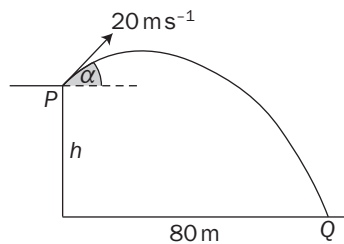


1. A golfer hits a ball from a point O on a horizontal surface. The initial velocity of the ball is 35 m s^{-1} at an angle of 40° to the horizontal.
- Calculate the maximum height reached by the ball.
 - Show that the range of the ball is 123 m.
 - Find the speed and direction of the ball 1.5 s after it leaves O .
 - What assumptions have you made in modelling this situation?

2. A particle P is projected with velocity $(u\mathbf{i} + 2u\mathbf{j}) \text{ m s}^{-1}$ from a point O on a horizontal plane, where \mathbf{i} and \mathbf{j} are horizontal and vertical unit vectors respectively. The particle P strikes the plane at the point A , where $OA = 156.8 \text{ m}$.
- Show that the time of flight is 8 seconds.
 - Find the maximum height reached by the particle.
- The particle P passes through a point B with speed 24.5 m s^{-1} .
- Find the height of B above the horizontal plane.

3.



A particle is projected with speed 20 m s^{-1} from a point P on a cliff above horizontal ground. The angle of projection is α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The ball hit the ground at a point Q , at a horizontal distance of 80 m from P , as shown in the diagram.

- Find the greatest height above P reached by the particle.
- Find the height of P above the ground.
- By considering energy, or otherwise, find the speed of the particle as it hits the ground at Q .

4. A particle P is projected horizontally from a point A with speed $V \text{ m s}^{-1}$. At the same moment a second particle Q is projected at an angle α above the horizontal with speed $U \text{ m s}^{-1}$ from a point B which is h m directly below A . The particles move freely under gravity in the same vertical plane and collide in mid-air.
- (a) By considering the horizontal motion of each particle, show that $\cos \alpha = \frac{V}{U}$
- (b) Hence state a relationship between U and V for a collision to be possible.
- (c) Given that $U = 25$ and $V = 20$, and that the particles collide 2 seconds after projection, find the value of h .

5. Gina is standing in a sports hall at a distance of 30 m from the end wall. The ceiling is 10 m above the floor. She kicks a ball from the floor at 20 m s^{-1} at an angle α above the horizontal.
- (a) If the ball just brushes the ceiling, find the value of α .
- (b) How far up the wall does the ball strike it?

6. A particle P travels along a straight line through a point O so that at time t s after passing through O its displacement from O is x m, where $x = 2t^3 - 15t^2 + 24t$
- (a) (i) Find an expression for its velocity, $v \text{ m s}^{-1}$, at time t s.
(ii) Hence find its initial velocity.
- (b) Find the times at which the particle is instantaneously at rest, and its position at those times.
- (c) Find the distance travelled by the particle in the first 5 s of the motion.
- (d) Find the value of t for which the particle has zero acceleration.

7. A particle P is moving along the x -axis. Its acceleration, $a \text{ m s}^{-2}$, at time t seconds is given by $a = (6t - 18)$. The velocity of P at time t seconds is $v \text{ m s}^{-1}$. Given that $v = 15$ when $t = 0$, find
- (a) v in terms of t
- (b) the distance between the two points where P is instantaneously at rest.

8. A particle P , of mass 2 kg, is moving in a plane under the action of a single force \mathbf{F} newtons. At time t seconds, its velocity, $\mathbf{v} \text{ m s}^{-1}$, is given by
- $$\mathbf{v} = (8t + 3)\mathbf{i} + 3t(t + 1)\mathbf{j}$$
- When $t = 0$, P is at the point A with position vector $(2\mathbf{i} + 3\mathbf{j})$ m.
When $t = 2$, P is at the point B . Calculate
- (a) the magnitude of \mathbf{F} when $t = 2$
- (b) the distance AB .

9. The velocity \mathbf{v} m s^{-1} of a particle P at time t seconds is given by

$$\mathbf{v} = (4t - 5)\mathbf{i} + 3t\mathbf{j}$$

- (a) (i) Show that the acceleration of P is constant.
(ii) Find the magnitude and direction of this acceleration.

At $t = 0$, P is at the point A , with position vector $(-3\mathbf{i} + 2\mathbf{j})$ m relative to a fixed origin O . When $t = 4$ the particle is at the point B .

- (b) Find an expression for the position vector of P at time t .
Hence find
(i) the time at which P crosses the y -axis
(ii) the position vector of B
(iii) the distance AB .

10. Two particles, P and Q , move in a plane. Initially P is at the origin and its velocity at time t s is $\mathbf{v}_P = ((2t + 3)\mathbf{i} + (6t^2 - 1)\mathbf{j})$ m s^{-1} . Q is initially stationary at the point with position vector $(2\mathbf{i} + k\mathbf{j})$ m, and its acceleration at time t s is

$$\mathbf{a}_Q = (4\mathbf{i} + 6t\mathbf{j}) \text{ m s}^{-2}$$

- (a) Find expressions for \mathbf{r}_P and \mathbf{r}_Q , the position vectors of P and Q at time t s.
(b) Given that the particles collide, find the two possible values of k .



1 Kinematics

Question Number	Solution	Marks
1 a	Considering vertical: $v^2 = u^2 + 2as$ $0 = (35 \sin 40^\circ)^2 - 19.6s$ $s = 25.8 \text{ m}$	M1 M1 A1
b	Using $v = u + at$ $t = \frac{35 \sin 40^\circ}{9.8} = 2.30 \text{ s}$ (3 sf) to max height so range in 4.59 s Considering horizontal: $s = ut$ $s = 35 \cos 40^\circ \times 4.59 = 123 \text{ m}$	M1 A1 M1 A1
c	Horizontal velocity = $35 \cos 40^\circ$ Vertical velocity = $u + at$ $v = 35 \sin 40^\circ - 9.8 \times 1.5$ $v = 7.80$ So speed = $\sqrt{(35 \cos 40^\circ)^2 + 7.80^2} = 27.9 \text{ m/s}$ $\theta = \tan^{-1}\left(\frac{7.80}{35 \cos 40^\circ}\right) = 16.2^\circ$	B1 M1 A1 M1 A1 M1 A1
d	The golf ball is a particle, no air resistance or other sensible comments.	B1
		15
2 a	Considering vertical distance travelled for range (=0): $s = ut + \frac{1}{2}at^2$ $0 = 2ut - 4.9t^2 = t(2u - 4.9t)$ $t = 0$ or $t = \frac{2u}{4.9} \Rightarrow u = 2.45t$ Substituting into horizontal: $156.8 = 2.45t \times t \Rightarrow t = 8 \text{ s}$	M1 M1 A1 M1 A1
b	Max height after 4 seconds Considering vertical: $s = 39.2 \times 4 - 4.9(4^2) = 78.4 \text{ m}$	B1 M1 A1
c	$u = 2.45 \times 8 = 19.6$ ($2u = 39.2$) Initial speed = $\sqrt{19.6^2 + 39.2^2} = 43.8 \text{ ms}^{-1}$ Considering energy: Initial KE = Final PE + Final KE: $\frac{1}{2}m(43.8)^2 = 9.8mh + \frac{1}{2}m(24.5)^2$ $h = 67.375 \text{ m} \approx 67.4 \text{ m}$	B1 M1 A1 M1 M1 A1
		14
© Oxford University Press 2009		Mechanics M2

<p>3 a</p>	<p>Considering vertical when $v = 0$:</p> $s = \frac{v^2 - u^2}{2a} = \frac{12^2}{19.6} = 7.35 \text{ m}$	<p>M1 M1 A1</p>
<p>b</p>	<p>Considering horizontal where $u = 20 \cos \alpha = 16$</p> $t = \frac{80}{16} = 5 \text{ s}$ <p>Considering vertical where $u = 20 \sin \alpha = 12$</p> $s = ut + \frac{1}{2}at^2 = 60 - 4.9 \times 25 = -62.5$ <p>So $h = 62.5 \text{ m}$</p>	<p>M1 M1 A1 M1 M1 A1</p>
<p>c</p>	<p>Initial KE = Final KE + Final PE</p> $\frac{1}{2}m(20)^2 = \frac{1}{2}mv^2 + m(9.8 \times -62.5)$ $v = 40.3 \text{ m s}^{-1}$	<p>M1 A1</p> <p style="text-align: right;">11</p>
<p>4 a</p>	<p>$S_A = Vt, S_B = Ut \cos \alpha$</p> <p>Equating the two: $Vt = Ut \cos \alpha \Rightarrow \cos \alpha = \frac{V}{U}$</p>	<p>M1 A1</p>
<p>b</p>	<p>$V \leq U$</p>	<p>B1</p>
<p>c</p>	<p>$\cos \alpha = \frac{4}{5}, \sin \alpha = \frac{3}{5}$</p> <p>Considering B, vertically, after 2 seconds:</p> $s = 25 \times 2 \sin \alpha - 4.9 \times 2^2 = 10.4 \text{ m}$ <p>Considering A, vertically, after 2 seconds, with initial vertical velocity = 0:</p> $s = 4.9 \times 2^2 = 19.6 \text{ m}$ <p>Total distance = 30 m</p>	<p>M1 M1 A1 A1</p> <p style="text-align: right;">9</p>
<p>5 a</p>	<p>Considering vertical:</p> $v^2 = u^2 + 2as$ $0 = (20 \sin \alpha)^2 - 19.6 \times 10 \Rightarrow \sin \alpha = \sqrt{\frac{196}{400}} \Rightarrow \alpha = 44.4^\circ$	<p>M1 M1 A1</p>
<p>b</p>	<p>Considering horizontal, $t = \frac{30}{20 \cos \alpha} = 2.1 \text{ s}$</p> <p>Considering vertical:</p> $s = ut + \frac{1}{2}at^2 = 20 \times 2.1 \sin \alpha - 4.9 \times 2.1^2 = 7.80 \text{ m}$	<p>M1 A1 M1 A1</p> <p style="text-align: right;">7</p>

<p>6 a</p> <p>b</p> <p>c</p> <p>d</p>	$v = \frac{dx}{dt} = 6t^2 - 30t + 24$ <p>At $t = 0$, $v = 24 \text{ m s}^{-1}$</p> $v = 0: 6t^2 - 30t + 24 = 0$ $0 = (t - 4)(t - 1) \Rightarrow t = 1 \text{ or } 4$ <p>At $t = 1$, $x = 11 \text{ m}$</p> <p>At $t = 4$, $x = -16 \text{ m}$</p> <p>From O to $t = 1$, distance is 11 m. From $t = 1$ to $t = 4$, distance is $11 + 16 = 27 \text{ m}$. At $t = 5$, $x = -5$ so distance from $t = 4$ to $t = 5$ is 11 m. Total distance = 49 m.</p> $a = \frac{dv}{dt} = 12t - 30 = 0$ <p>$t = 2.5 \text{ s}$</p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>M2</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">12</p>
<p>7 a</p> <p>b</p>	$v = \int (6t - 18) dt = 3t^2 - 18t + c$ <p>$t = 0$, $v = 15 \Rightarrow c = 15$ Hence $v = 3t^2 - 18t + 15$</p> $v = 0 \Rightarrow 3t^2 - 18t + 15 = 0$ $0 = (t - 5)(t - 1)$ <p>$t = 1$ or $t = 5$</p> $x = \int v dt = t^3 - 9t^2 + 15t (+c (=0))$ <p>$t = 5$, $x = -25$</p> <p>$t = 1$, $x = 7$</p> <p>So distance between is 32 m.</p>	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">8</p>
<p>8 a</p> <p>b</p>	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 8\mathbf{i} + (6t + 3)\mathbf{j}$ $\mathbf{F} = m\mathbf{a} = 2(8\mathbf{i} + (6 \times 2 + 3)\mathbf{j}) = 16\mathbf{i} + 30\mathbf{j}$ $ \mathbf{F} = \sqrt{16^2 + 30^2} = 34 \text{ N}$ $\mathbf{r} = \int \mathbf{v} dt + (2\mathbf{i} + 3\mathbf{j}) = (4t^2 + 3t + 2)\mathbf{i} + \left(t^3 + \frac{3}{2}t^2 + 3\right)\mathbf{j}$ <p>$t = 2$, $\mathbf{r} = 24\mathbf{i} + 17\mathbf{j}$</p> $ \mathbf{AB} = 22\mathbf{i} + 14\mathbf{j} = \sqrt{22^2 + 14^2} = 26.1 \text{ m}$	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p style="text-align: right;">11</p>
<p>9 a</p> <p>b</p>	<p>i $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 4\mathbf{i} + 3\mathbf{j} = \text{constant}$</p> <p>ii $\mathbf{a} = 5 \text{ m s}^{-1}$</p> <p>$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$ to positive \mathbf{i}</p> $\mathbf{r} = \int \mathbf{v} dt + (-3\mathbf{i} + 2\mathbf{j}) = (2t^2 - 5t - 3)\mathbf{i} + \left(\frac{3}{2}t^2 + 2\right)\mathbf{j}$ <p>i Solve \mathbf{i} component = 0:</p> $2t^2 - 5t - 3 = 0$ $0 = (2t + 1)(t - 3)$ <p>$t = 3$ seconds (only positive solution required)</p> <p>ii $t = 4$, $\mathbf{r} = 9\mathbf{i} + 26\mathbf{j}$</p> <p>iii $\mathbf{AB} = 12\mathbf{i} + 24\mathbf{j} = \sqrt{12^2 + 24^2} = 26.8 \text{ m}$</p>	<p>M1 A1</p> <p>A1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p style="text-align: right;">12</p>

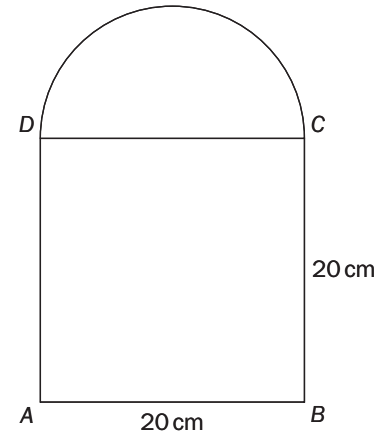
<p>10 a</p> <p>b</p>	<p>$\mathbf{r}_p = \int \mathbf{v}_p dt = (t^2 + 3t)\mathbf{i} + (2t^3 - t)\mathbf{j}$ since it starts at the origin</p> <p>$\mathbf{v}_Q = \int \mathbf{a}_Q dt = 4t\mathbf{i} + 3t^2\mathbf{j}$ since initially at rest</p> <p>$\mathbf{r}_Q = \int \mathbf{v}_Q dt = 2t^2\mathbf{i} + t^3\mathbf{j} + (2\mathbf{i} + k\mathbf{j})$</p> <p>Equate i components of the two expressions for r:</p> <p>$t^2 + 3t = 2t^2 + 2$ $0 = t^2 - 3t + 2$ $0 = (t - 1)(t - 2)$</p> <p>Substitute values into j components to solve for k:</p> <p>$t = 1 \Rightarrow t^3 + k = 1$ so $k = 0$ $t = 2 \Rightarrow t^3 + k = 14$ so $k = 6$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 A1</p> <p style="text-align: right;">11</p>

Centre of mass

1. A uniform metal plate comprises a square lamina $ABCD$ attached along CD to the diameter of a semicircular lamina, as shown in the diagram.
- (a) Taking A as the origin and AB and AD as the coordinate axes, find the coordinates of the centre of mass of the plate.

The plate is now suspended from the point D and hangs freely in equilibrium.

- (b) Find the angle between the side AB and the vertical.



- 2.

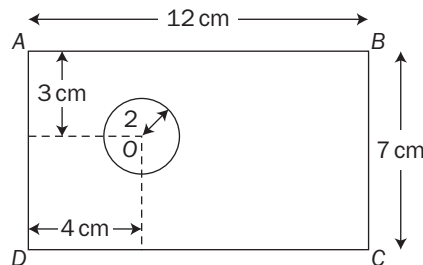


Figure 1

Figure 1 shows the cross-section of a metal block made by drilling a circular hole of centre O and radius 2 cm through a uniform cuboid. $ABCD$ is the end of the cuboid, where $AB = 12$ cm and $BC = 7$ cm. The point O is 4 cm from both CD and AD .

- (a) Calculate, to 3 significant figures, the distance of the centre of mass of the plate
- (i) from AD (ii) from CD .

The block is placed on a rough plane inclined at an angle α to the horizontal, with DA in contact with the plane and in the direction of the line of greatest slope, as shown in Figure 2.

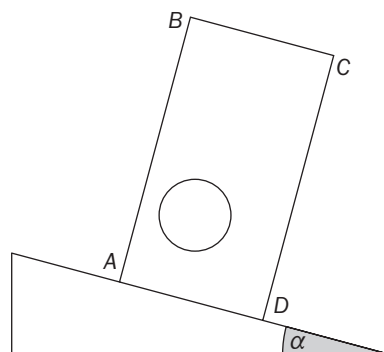
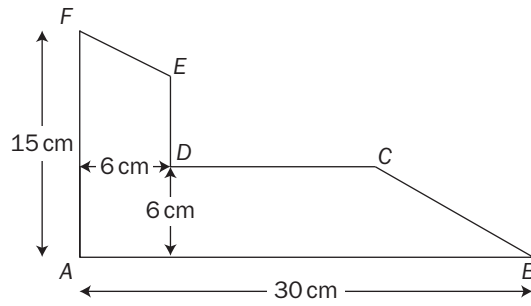


Figure 2

- (b) Assuming that the coefficient of friction is large enough to prevent the block from slipping, calculate, to the nearest degree, the value of α for which the block is on the point of toppling over.

3.



The diagram shows a carpenter's tri-square, comprising a uniform triangular lamina ABF from which a triangle DCE has been removed.

$AB = 30$ cm, $AF = 15$ cm and the blades of the tri-square are 6 cm wide, as shown.

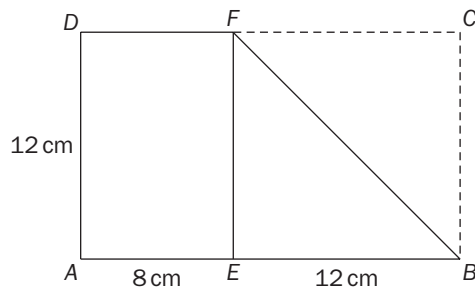
(a) Find the distance of the centre of mass of the tri-square from

- (i) AF (ii) AB .

The tri-square is designed to hang up for storage, suspended from the corner F .

(b) If the tri-square is hanging freely in equilibrium, calculate, to the nearest degree, the angle between AF and the vertical.

4.



A uniform rectangular lamina $ABCD$, where $AB = 20$ cm and $AD = 12$ cm, is folded as shown so that the corner C rests at the point E on AB .

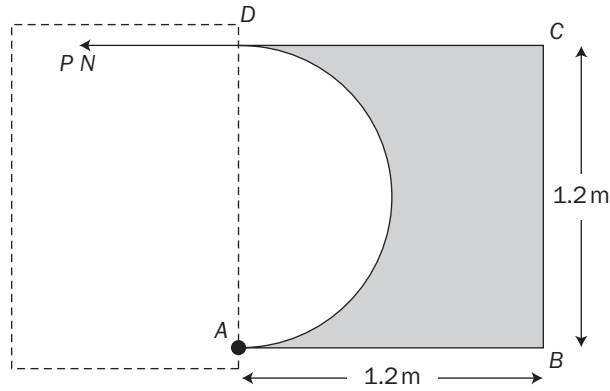
(a) Find the distance of the centre of mass of the folded lamina

- (i) from AD
(ii) from AB .

The mass of the folded lamina is m kg. A particle of mass km kg is attached to the lamina at D and the combined object is freely suspended in equilibrium from A .

(b) If the point F hangs vertically below A , show that $k = 0.7$.

5.



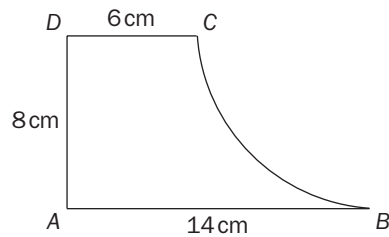
A shop sign is a uniform lamina formed by taking a uniform square sheet of metal $ABCD$, of side 1.2 m , and removing the semi-circle with diameter AD , as shown in the diagram.

- (a) Find, to 3 significant figures, the distance of the centre of mass of the lamina from the mid-point of AD .

The sign has mass 5 kg . It is erected by freely hinging it at A and holding it with BC vertical by means of a horizontal tie rod at D , as shown. The tension in the tie rod is $P\text{ N}$.

- (b) Find (i) the value of P
(ii) the magnitude and direction of the force at the hinge.

6.



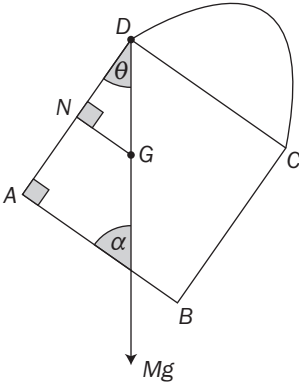
A thin uniform wire is bent to form the framework shown, with three straight rectilinear edges $AB = 14\text{ cm}$, $CD = 6\text{ cm}$ and $AD = 8\text{ cm}$. BC is a quarter circle.

- (a) Taking AB and AD as the x - and y -axes, find the coordinates of the centre of mass of this framework.

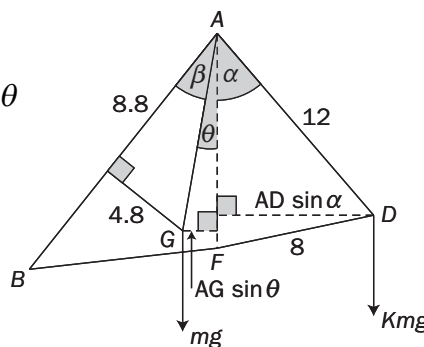
The mass of the framework is M . A particle of mass kM is attached to the framework at B so that when the framework is freely suspended in equilibrium from the mid-point of AB , the side AB is horizontal.

- (b) Find the value of k .

Centre of mass

Question Number	Solution	Marks
<p>1 a</p> <p>b</p>	<p>$\bar{x} = 10$ cm by symmetry</p> <p>Taking moments about AB:</p> <p>Mass of square: 400, mass of semicircular sector: 157.1</p> <p>Semicircles centre of mass $\frac{4r}{3\pi} =$ from centre 4.244 (+20)</p> <p>$400 \times 10 + 157.1 \times 24.244 = \bar{y} \times 557.1$</p> <p>$\bar{y} = 14.02$ cm</p> <p>$\tan \alpha = (90^\circ - \theta)$</p> <p>$= \cot \theta$</p> <p>$= \frac{DN}{NG}$</p> <p>$= \frac{20 - 14.02}{10}$</p> <p>$\alpha = \tan^{-1}\left(\frac{20 - 14.02}{10}\right) = 30.9^\circ$</p> 	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 (numerator)</p> <p>M1 A1</p> <p style="text-align: right;">8</p>
<p>2 a</p> <p>b</p>	<p>Mass of block: 84, mass of circle: 4π</p> <p>i Taking moments about AD:</p> <p>$84 \times 6 - 4\pi \times 4 = \bar{x}(84 - 4\pi)$</p> <p>$\bar{x} = 6.35$ cm</p> <p>ii Taking moments about DC:</p> <p>$84 \times 3.5 - 4\pi \times 4 = \bar{y}(84 - 4\pi)$</p> <p>$\bar{y} = 3.41$ cm</p> <p>Centre of mass acts over corner D:</p> <p>$\alpha = \tan^{-1}\left(\frac{3.41}{6.35}\right) = 28^\circ$ (nearest degree)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p style="text-align: right;">7</p>

<p>3 a</p>	<p>Centre of mass of triangle: $\frac{2}{3}$ along the median from the vertex</p> <p>Big: 10 cm from AF, 5 cm from AB</p> <p>Little: 10 cm from AF, 8 cm from AB</p> <p>i By symmetry, since both centres act 10 cm from AF, centre of mass of compound lamina is 10 cm from AF.</p> <p>ii Taking moments about AB:</p> $225 \times 5 - 36 \times 8 = 189\bar{y}$ $\bar{y} = 4.43 \text{ cm}$	<p>M1</p> <p>M1 A1</p> <p>B1 ft</p> <p>M1</p> <p>A1</p> <p>M1 (denom)</p> <p>M1 A1</p> <p style="text-align: right;">9</p>
<p>4 a</p>	<p>Rectangle mass = 96, triangle mass = $2 \times 72 = 144$</p> <p>Rectangle centre = (4, 6), triangle centre = (12, 4)</p> <p>Taking moments about AD and AB:</p> $96 \times 4 + 144 \times 12 = 240\bar{x}$ $\bar{x} = 8.8 \text{ cm}$ $96 \times 6 + 144 \times 4 = 240\bar{y}$ $\bar{y} = 4.8 \text{ cm}$ <p>b Taking moments about A:</p> $k mg \times AD \sin \alpha = mg \times AG \sin \theta$ <p>Therefore $k \times 12 \sin \alpha = \sqrt{4.8^2 + 8.8^2} \sin \theta$</p> <p>Now $\tan \alpha = \frac{8}{12}$ so $\alpha = 33.7^\circ$</p> <p>and $\theta = 90^\circ - \alpha - \beta$</p> $= 90^\circ - 33.7^\circ - \tan^{-1}\left(\frac{4.8}{8.8}\right)$ $= 90^\circ - 33.7^\circ - 28.6^\circ$ $= 27.7^\circ$ <p>Therefore $k \times 12 \sin 33.7^\circ = 10.02 \sin 27.7^\circ$</p> <p>Therefore $k = \frac{4.6577}{6.6581} = 0.6995 \dots \approx 0.7$</p>	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>
<p>5 a</p>	<p>Mass of square: 1.44, mass of semicircle: 0.18π</p> <p>Centre for square: 0.6, centre for semicircle: 0.2546...</p> <p>Taking moments about AD:</p> $1.44 \times 0.6 - 0.18\pi \times 0.2546 = (1.44 - 0.18\pi)\bar{x}$ $\bar{x} = 0.823 \text{ m}$ <p>b i Taking moments about A:</p> $1.2P = 5g \times 0.823$ $P = 33.6 \text{ N}$ <p>ii Magnitude: $\sqrt{33.6^2 + (5g)^2} = 59.4 \text{ N}$</p> <p>Direction: $\tan^{-1}\left(\frac{5g}{33.6}\right) = 55.6^\circ$ relative to +ve AB</p>	<p>B1 B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p style="text-align: right;">12</p>
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<p>6 a</p>	<p>Centre of mass of arc: $\left(\frac{r \sin \alpha}{\alpha}\right) = \frac{8 \sin \frac{\pi}{4}}{\frac{\pi}{4}} = 7.202530529$</p> <p>from the centre so from corner 'E' $\sqrt{\frac{1}{2}(7.202530529)^2} = 5.09...$ in both x and y direction.</p> <p>Taking moments about AD and AB:</p> <p>$14 \times 7 + 6 \times 3 + 4\pi(14 - 5.09...) = \bar{x}(28 + 4\pi)$ $\bar{x} = 5.62 \text{ cm}$</p> <p>$8 \times 4 + 6 \times 8 + 4\pi(8 - 5.09...) = \bar{y}(28 + 4\pi)$ $\bar{y} = 2.87 \text{ cm}$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 M1 A1</p>
<p>b</p>	<p>$kM \times 7 - (7 - 5.62)M = 0$ $k = 0.197$</p>	<p>M1 M1 A1</p>
11		



3

Work, energy and power

1. Noah slides a small block of mass 1 kg directly up a rough plank inclined at an angle α to the horizontal. The coefficient of friction between the block and the plank is 0.2.

Initially, $\alpha = 10^\circ$ and he slides the block with an initial speed of $v \text{ ms}^{-1}$.

It first comes to rest after travelling 3 m.

- (a) Find (i) the work done against friction
(ii) the value of v .

α is now increased to 20° and Noah slides the block with initial speed 10 ms^{-1} .

- (b) Find how far the block travels before coming to rest.

2. A car of mass 1200 kg tows a trailer of mass 300 kg along a straight horizontal road. The resistance to motion is modelled as being a constant force of magnitude 800 N acting on the car and a constant force of magnitude 200 N acting on the trailer. The power generated by the engine is 30 kW.

- (a) Calculate (i) the acceleration when the car is travelling at 12 m s^{-1}
(ii) the maximum speed of the car and trailer.

When the car is travelling at 15 m s^{-1} , the trailer breaks loose.

Assuming the same model for resistance

- (b) use the work-energy principle to calculate the distance the trailer travels before coming to rest.
(c) Give a reason why the model used for the resistance to motion may not be realistic.

3. A car of mass 1000 kg is driving up a straight road inclined at an angle of 5° to the horizontal. The resistance to motion is modeled as a constant force of magnitude 1000 N.

Given that initially the speed of the car is 10 m s^{-1} and that the car's engine generates a constant power of 24 kW

- (a) calculate the magnitude of the initial acceleration of the car.

When travelling down the same hill, the rate that the car's engine is working is reduced to 5 kW. Using the same model for the resistance

- (b) calculate in m s^{-1} the maximum speed of the car.

If the car had continued working at the previous rate, the model predicts that the maximum speed down the hill would be nearly 600 km h^{-1} , which is unrealistic.

- (c) Explain what aspect of the model you would need to change to give a more realistic prediction.

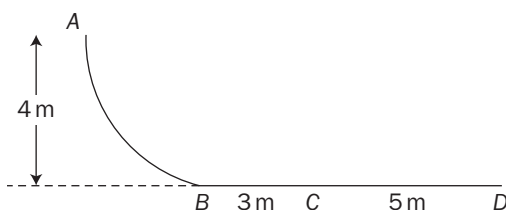
4. A car of mass 800 kg is moving at a constant speed of 54 km h^{-1} up a slope inclined at an angle α to the horizontal. The power generated by the engine is a constant 18 kW and the resistance to motion is modelled as a constant force of magnitude 400 N .

(a) Find the value of α .

While the car is travelling up the slope at 54 km h^{-1} , the driver puts her foot on the clutch and the car coasts to rest.

(b) Assuming the resistance force is unchanged, find how far the car travels before coming to rest.

5.



The diagram shows a slide in a playground. It consists of a curved slope AB , with A at a height of 4 m above B , and a horizontal section BCD , where $BC = 3\text{ m}$ and $CD = 5\text{ m}$. AB and BC are smooth, while CD is rough with coefficient of friction μ .

Frances has mass $m\text{ kg}$. She starts from rest at A . Find

(a) Frances's speed when she reaches C

(b) the minimum value of μ for which Frances will not go beyond the end of the slide at D .

6. Binky is cycling at a constant speed of 6 m s^{-1} up a slope inclined at 3° to the horizontal. Binky and her bicycle have a combined mass of 50 kg . The resistance to motion from non-gravitational forces is modelled as being constant, with magnitude 15 N .

(a) Show that her rate of working is 244 W .

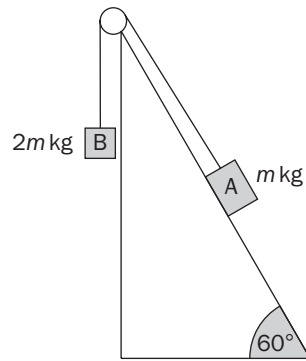
(b) She now free-wheels from rest for a distance of 100 m down the same slope. Assuming the same model for resistance, find her speed at the end of the 100 m slope.

On another occasion she pedals down the same slope. Her rate of work is again 244 W but she is wearing rain clothing, so the resistance to motion is increased to 35 N .

(c) Find her maximum speed as predicted by the model of constant resistance.

(d) Would you expect her actual final speed to be higher or lower than that predicted in part c? Explain your answer.

7.



Two particles A and B , of mass m kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. Particle A lies on a rough plane inclined at 60° to the horizontal. The string passes over a small light smooth pulley at the top of the plane. The particle B hangs freely, as shown in the diagram.

The particles are released from rest with the string taut. The coefficient of friction between A and the plane is 0.5 . When each particle has moved a distance h m, B has not reached the ground and A has not reached the pulley.

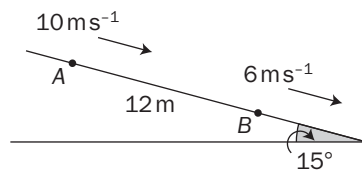
(a) Find an expression for

- (i) the gain in potential energy by particle A in moving a distance h
- (ii) the loss of potential energy by particle B in moving a distance h
- (iii) the work done against friction in moving A a distance h .

When each particle has moved a distance h , they are moving with speed v .

(b) Using the work-energy principle, find an expression for v^2 in terms of g and h .

8.



A block of mass 4 kg is sliding down a rough plane inclined at 15° to the horizontal. The coefficient of friction between the block and the plane is μ . The block slides down a line of greatest slope of the plane from a point A to a point B , where $AB = 12$ m. At A the block is moving at 10 m s^{-1} and when it reaches B its speed has reduced to 6 m s^{-1} , as shown in the diagram.

- (a) Find
- (i) the total energy lost by the block in travelling from A to B
 - (ii) the value of μ .
- (b) State two modelling assumptions that you have made.



3 Work, energy and power

Question Number	Solution	Marks
1 a	<p>i Work done against friction = $\mu N \times 3$ $= \mu g \cos 10^\circ \times 3$ $= 0.2g \cos 10^\circ \times 3 = 5.79\text{J}$</p> <p>ii Work done by friction = (KE + PE) end - (K + PE) start i.e. $-5.79 = 3g \sin 10^\circ - \frac{1}{2}v^2$</p> <p>So $\frac{1}{2}v^2 = 5.79 + 3g \sin 10^\circ$ $= 10.89$ and $v = 4.67 \text{ m s}^{-1}$</p>	M1 A1 M1 A1
b	<p>Total force in direction of motion: $-(0.2g \cos 20^\circ + g \sin 20^\circ) = -5.19 \text{ N}$ Work done = $5.19x$</p> <p>Loss in KE = $\frac{1}{2}mv^2 = 50\text{J}$</p> <p>$x = \frac{50}{5.19} = 9.63 \text{ m}$</p>	M1 A1 M1 M1 M1 A1
2 a	<p>i Power = Dv $30000 = 12D$ $D = 2500 \text{ N}$ $F = ma: 2500 - 1000 = 1500a$ $a = 1 \text{ m s}^{-2}$</p> <p>ii Max speed when $D = 1000 \text{ N}$ $30000 = 1000v$ $v = 30 \text{ m s}^{-1}$</p>	M1 A1 M1 A1 M1 M1 A1
b	<p>Work done by force = loss in KE</p> <p>$200x = \frac{1}{2}(300)(15)^2$</p> <p>$x = 168.75 \text{ m}$</p>	M1 A1
c	It may not be constant resistance.	B1
		12
		10

<p>3 a</p> <p>Power = Dv $24\,000 = 10D$ so $D = 2400\text{ N}$</p> $F = ma \Rightarrow a = \frac{2400 - 1000 - 1000g \sin 5^\circ}{1000} = 0.546\text{ m s}^{-1}$ <p>b</p> $D = 1000 - 1000g \sin 5^\circ = 145.87$ $5000 = 145.87v$ $v = 34.28\text{ m s}^{-1}$ <p>c</p> <p>Variable resistance force proportional to speed</p>		<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p style="text-align: right;">9</p>
<p>4 a</p> <p>Speed = 15 m s^{-1} $18\,000 = 15D$ so $D = 1200\text{ N}$ $1200 - 400 - 800g \sin \alpha = 0$ $\alpha = 5.86^\circ$</p> <p>b</p> $-(400 + 800g \sin \alpha) = 800a$ $a = -1.5\text{ m s}^{-1}$ <p>Using $v^2 = u^2 + 2as$: $0 = 225 - 3s$ $s = 75\text{ m}$</p>		<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">9</p>
<p>5 a</p> <p>Loss in PE = Gain in KE</p> $4mg = \frac{1}{2}mv^2$ $v^2 = 8g$ so $v = 8.85\text{ m s}^{-1}$ <p>b</p> <p>Must lose $39.2m$ joules in 5 metres (from a) Work done by force = $5 \times \mu R = 5\mu mg = 39.2m$</p> $\mu = \frac{39.2}{5g} = 0.8$		<p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">5</p>
<p>6 a</p> $D = 15 + 50g \sin 3^\circ = 40.64\text{ N}$ $P = Dv = 40.64 \times 6 = 244\text{ W (3 sf)}$ <p>b</p> <p>Work done = Gain in KE</p> $100(50g \sin 3^\circ - 15) = \frac{1}{2}(50)v^2$ $v = 6.53\text{ m s}^{-1}$ <p>c</p> $D + 50g \sin 3^\circ - 35 = 0$ $D = 9.355\text{ N}$ $P = Dv \Rightarrow 244 = 9.355v \Rightarrow v = 26.1\text{ m s}^{-1}$ <p>d</p> <p>Lower (since resistance would increase proportional to speed).</p>		<p>M1 A1</p> <p>M1 A1 (cao)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>E1</p> <p style="text-align: right;">11</p>

<p>7 a</p> <p>i $mgx = mgh \sin 60^\circ = \frac{\sqrt{3}}{2}mgh$</p> <p>ii $2mgh$</p> <p>iii $F = \mu R \Rightarrow F = 0.5mg \cos 60^\circ = \frac{1}{4}mg$</p> <p>Work done = $\frac{1}{4}mgh$</p> <p>b</p> <p>Kinetic Energy in the system = $\frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2$</p> <p>Gain in KE = Loss in PE(B) – gain in PE(A) – Work done</p> <p>$\frac{3}{2}mv^2 = 2mgh - \frac{\sqrt{3}}{2}mgh - \frac{1}{4}mgh$</p> <p>$v^2 = \frac{7 - 2\sqrt{3}}{6}gh$</p>		<p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">11</p>
<p>8 a</p> <p>i Loss in energy = Initial KE + Initial PE – Final KE</p> <p>$\frac{1}{2}(4)(10)^2 + 4g \times 12 \sin 15^\circ - \frac{1}{2}(4)(6)^2 = 249.75\text{J}$</p> <p>ii Work done = Fx</p> <p>$F = \frac{249.75}{12} = 20.8\text{N}$</p> <p>$F = \mu R \Rightarrow 20.8 = \mu \times 4g \cos 15^\circ$</p> <p>$\mu = 0.55$</p> <p>b</p> <p>Block is a particle, constant resistive force, no air resistance... (any two sensible assumptions)</p>		<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>E1</p> <p style="text-align: right;">8</p>



4

Collisions

1. A particle A , of mass 2 kg , is moving with speed 6 m s^{-1} on a smooth horizontal surface when it collides directly with a second particle B , of mass 4 kg , moving with speed 3 m s^{-1} in the same direction as A . The coefficient of restitution between A and B is $\frac{1}{2}$.

- (a) Find the speed and direction of both particles after the collision.
(b) State the magnitude of the impulse received by B during the collision.

B goes on to collide directly with a third particle C , of mass 1 kg , which is at rest. The coefficient of restitution between B and C is e . There are no further collisions.

- (c) Find the range of possible values for e .

2. A smooth sphere A , of mass m , is moving in a straight line with speed $3u$ on a smooth horizontal surface. A second smooth sphere B , of mass $2m$, is travelling in the same direction with speed u . Sphere A collides directly with B . The coefficient of restitution between A and B is 0.5 . The spheres are modelled as particles.

- (a) Show that B 's speed is doubled by the collision, and find the new velocity of A .

After the collision, B strikes a fixed vertical wall which is perpendicular to the direction of motion of A and B . The coefficient of restitution between B and the wall is 0.8 . Spheres A and B then collide again.

- (b) Find the velocities of A and B after they collide for the second time.
(c) Find the kinetic energy lost during the whole sequence of collisions.

3. Three particles A , B and C lie at rest, in that order, in a straight line on a smooth horizontal surface. Their masses are $2m$, m and km respectively. Particle A is projected towards B with speed $3u$, and simultaneously particle B is projected towards A with speed u . Particles A and B collide. The coefficient of restitution between A and B is 0.8 .

- (a) Find the speed and direction of the two particles after this collision.
(b) Particle B now collides with C with a coefficient of restitution of 0.5 . Find the new velocity of C in terms of u and k .
(c) Given that A and B collide for a second time, find the range of possible values of k .

4. A particle A , of mass $3m$, is moving with speed u in a straight line on a smooth horizontal surface. The particle collides directly with a second particle B , of mass $2m$, moving with speed u in the opposite direction to A . Immediately after the collision the speed of B is $0.8u$ and its direction of motion is reversed.

- (a) Calculate the coefficient of restitution between A and B .
(b) Show that the kinetic energy lost in the collision is $1.8mu^2$.

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of A and B . The coefficient of restitution in this collision is e . There are then no more collisions.

- (c) Find the range of possible values of e .
(d) Given that the magnitude of the impulse of the wall on B is $1.92mu$, find the exact value of e .

5. \mathbf{i} and \mathbf{j} are perpendicular unit vectors in a horizontal plane. An ice hockey puck, of mass 0.1 kg, is moving with velocity $(20\mathbf{i})\text{ m s}^{-1}$ when it is struck by a stick, which exerts an impulse of $(-1.72\mathbf{i} + 0.96\mathbf{j})\text{ N s}$ on the puck.

Find

- (a) the speed of the puck immediately after the impact
(b) the angle through which the puck is deflected as a result of the impact
(c) the kinetic energy lost by the puck in the impact.

The puck then collides directly with a second puck, also of mass 0.1 kg, which is at rest on the ice. After the collision both pucks move in the same direction with speeds in the ratio $1:3$.

- (d) Find the coefficient of restitution between the pucks.

6. Two particles, A and B , have masses $3m$ and $5m$ respectively. Particle A is moving with speed $4u$ on a smooth horizontal surface when it collides directly with B , which is at rest. The coefficient of restitution between A and B is e .

- (a) Show that the speed of B immediately after the collision is $\frac{3u(1+e)}{2}$ and find the speed of A .

B goes on to hit a smooth vertical wall which is perpendicular to the direction of motion of A and B . The coefficient of restitution between B and the wall is f .

- (b) Show that whatever the value of f , there is a second collision between A and B provided $e < 0.6$.
(c) Given that there is a second collision between A and B , even though $e = 0.8$, find the range of possible values of f .

7. Three particles A , B and C have masses of $2m$, m and m respectively. They are at rest, in a straight line and in the order A , B , C , on a smooth horizontal surface.

The coefficient of restitution between each pair of particles is 0.5 . Particles A and C are set in motion towards B , each with speed $2u$. A strikes B first.

- (a) Show that there will be three collisions, after which particle A will be at rest.
(b) Find the total loss of kinetic energy during the three collisions.

8. Three particles A , B and C lie at rest in a straight line on a smooth horizontal surface. Particle B is between A and C . The particles A and C each have mass $2m$, and the mass of B is m . Particle B is projected towards C with speed u . The coefficient of restitution between each pair of spheres is e . Take the direction ABC as positive.

(a) Show that, after B and C have collided, C has velocity $\frac{u(1+e)}{3}$ and find the velocity of B .

(b) Hence show that there is a collision between B and A provided $e > 0.5$.

Given that $e = 0.8$,

(c) show that after B and A have collided, there are no further collisions

(d) find the total loss of kinetic energy during the whole process.



4

Collisions

Question Number	Solution	Marks
<p>1 a</p> <p>b</p> <p>c</p>	<p>Using conservation of momentum and law of restitution: $6 \times 2 + 4 \times 3 = 2v_A + 4v_B \Rightarrow v_A + 2v_B = 12$</p> $\frac{1}{2}(6 - 3) = v_B - v_A \Rightarrow v_B = 1.5 + v_A$ <p>so $12 = v_A + 2(1.5 + v_A)$ $v_A = 3 \text{ m s}^{-1}$, $v_B = 4.5 \text{ m s}^{-1}$ both in original direction</p> <p>Impulse = change in momentum: $4.5 \times 4 - 3 \times 4 = 6 \text{ N s}$</p> <p>Conditions for no further collisions: $v_C > v_B \geq 3$ where v_B is now the velocity of B after the 2nd collision</p> $e = \frac{v_C - v_B}{4.5} \Rightarrow v_C = 4.5e + v_B$ $4.5 \times 4 = 4v_B + v_C = 4v_B + 4.5e + v_B$ $v_B = 3.6 - 0.9e \text{ and therefore } v_C = 3.6e + 3.6$ <p>Need $3 \leq v_B \leq v_C$</p> <p>i.e. $3 \leq 3.6 - 0.9e \leq 3.6 + 3.6e$</p> <p>$\therefore$ Need $0.9e \leq 0.6$ and $0 \leq 4.5e$</p> <p>Need $e \leq \frac{2}{3}$ and $e \geq 0$</p> <p>i.e. $0 \leq e \leq \frac{2}{3}$</p>	<p>M1 A1 (or equiv) M1 A1 (or equiv)</p> <p>M1 A1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (or equiv)</p> <p>M1 A2 (or equiv)</p> <p style="text-align: right;">18</p>
<p>2 a</p> <p>b</p> <p>c</p>	<p>Using conservation of momentum and law of restitution: $3mu + 2mu = mv_A + 2mv_B \Rightarrow v_A + 2v_B = 5u$</p> $\frac{1}{2}(3u - u) = v_B - v_A \Rightarrow v_B = u + v_A$ <p>so $5u = v_A + 2(u + v_A)$ $v_A = u$, $v_B = 2u$</p> <p>Speed of B after colliding with wall: $2u \times 0.8 = 1.6u$ towards A</p> $mu - 3.2mu = mv_A + 2mv_B \Rightarrow v_A + 2v_B = -2.2u$ $\frac{1}{2}(u - (-1.6u)) = v_B - v_A \Rightarrow v_B + 1.3u = v_A$ <p>so $-2.2u = v_A + 2(1.3u + v_A)$</p> $v_A = -1.6u$, $v_B = -0.3u$ <p>KE before first collision: $\frac{1}{2}m(3u)^2 + \frac{1}{2}(2m)u^2 = \frac{11}{2}mu^2$</p> <p>KE after third collision: $\frac{1}{2}m(1.6u)^2 + \frac{1}{2}(2m)(0.3u)^2 = 1.37mu^2$</p> <p>KE lost: $4.13mu^2$</p>	<p>M1 A1 (or equiv) M1 A1 (or equiv) M1 A1 A1</p> <p>M1 A1</p> <p>M1 A1 (or equiv)</p> <p>M1 A1 (or equiv) M1 A1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">21</p>

<p>3 a</p> <p>b</p> <p>c</p>	<p>Using conservation of momentum and law of restitution: $6mu - mu = 2mv_A + mv_B \Rightarrow 2v_A + v_B = 5u$</p> <p>$0.8(3u - (-u)) = v_B - v_A \Rightarrow v_B = 3.2u + v_A$ so $5u = 2v_A + 3.2u + v_A$ $v_A = 0.6u, v_B = 3.8u$</p> <p>$3.8mu = mv_B + kmv_C \Rightarrow v_B + kv_C = 3.8u$</p> <p>$0.5(3.8u) = v_C - v_B \Rightarrow v_B = v_C - 1.9u$</p> <p>so $3.8u = v_C - 1.9u + kv_C$ $v_C = \frac{5.7u}{1+k}$</p> <p>Condition for further collision: $v_B < 0.6u$ $\frac{5.7u}{1+k} - 1.9u < 0.6u$ $5.7u - 1.9u(1+k) < 0.6u(1+k) \Rightarrow k > 1.28$</p>	<p>M1 A1 (or equiv) M1 A1 (or equiv) M1 A1 A1</p> <p>M1 A1 (or equiv) M1 A1 (or equiv)</p> <p>M1 A1</p> <p>B1 M1 M1 A1</p> <p style="text-align: right;">17</p>
<p>4 a</p> <p>b</p> <p>c</p> <p>d</p>	<p>Using conservation of momentum and law of restitution: $3mu - 2mu = 3mv_A + 2m \times 0.8u \Rightarrow 3v_A = -0.6u$</p> <p>$e(u - (-u)) = 0.8u - v_A \Rightarrow v_A = 0.8u - 2ue$ so $-0.6u = 3(0.8u - 2ue)$ $e = 0.5$</p> <p>$v_A = -0.2u$</p> <p>KE before collision: $\frac{1}{2}(3m)u^2 + \frac{1}{2}(2m)u^2 = \frac{5}{2}mu^2$</p> <p>KE after collision: $\frac{1}{2}(3m)(0.2u)^2 + \frac{1}{2}(2m)(0.8u)^2 = 0.7mu^2$</p> <p>KE lost is therefore: $1.8mu^2$</p> <p>$v_B \leq 0.2u$ so $0.2u$ is the maximum value $e = \frac{v_B}{u_B} \Rightarrow 0.8ue \leq 0.2u \Rightarrow e \leq 0.25$</p> <p>Impulse = change in momentum: $2m \times 0.8u - 2mv_B = 1.92mu$ $v_B = -0.16u \Rightarrow e = \frac{0.16u}{0.8u} = 0.2$</p>	<p>M1 A1 (or equiv) M1 A1 (or equiv) M1 A1</p> <p>B1 ft M1 A1</p> <p>M1 A1 A1</p> <p>B1 M1 A1</p> <p>M1 M1 A1</p> <p style="text-align: right;">18</p>

<p>5 a</p> <p>Momentum before + Impulse = Momentum after $0.1 \times 20\mathbf{i} + (-1.72\mathbf{i} + 0.96\mathbf{j}) = 0.1\mathbf{v}$ $\mathbf{v} = (2.8\mathbf{i} + 9.6\mathbf{j}) \text{ m s}^{-1}$ Speed = $\sqrt{2.8^2 + 9.6^2} = 10 \text{ m s}^{-1}$</p> <p>b</p> <p>$\tan^{-1}\left(\frac{9.6}{2.8}\right) = 73.7^\circ$ to positive \mathbf{i}</p> <p>c</p> <p>$\frac{1}{2}(0.1)(20)^2 - \frac{1}{2}(0.1)(10)^2 = 15\text{J}$</p> <p>d</p> <p>Using conservation of momentum and law of restitution: $0.1 \times 10 = 0.1v_A + 0.1v_B \Rightarrow 1 = 0.4v_A$ since $3v_A = v_B$ $\Rightarrow v_A = 2.5$ $10e = v_B - v_A$ and $3v_A = v_B$ so $2v_A = 10e$ $e = 0.5$</p>	<p>M1 A1 M1 A1 M1 A1 M1 A1 M1 M1 A1 M1 A1</p> <p style="text-align: right;">13</p>
<p>6 a</p> <p>Using conservation of momentum and law of restitution: $3m \times 4u = 3mv_A + 5mv_B$ $v_B - v_A = 4ue \Rightarrow v_A = v_B - 4ue$ $12u = 3(v_B - 4ue) + 5v_B$ $v_B = \frac{3u(1+e)}{2}$ $v_A = \frac{3u + 3ue}{2} - 4ue = \frac{u(3-5e)}{2}$</p> <p>b</p> <p>$v_A > 0$ so $3 - 5e > 0 \Rightarrow e < 0.6$</p> <p>c</p> <p>$v_A = -0.5u, v_B = 2.7u$ $2.7uf > 0.5u \Rightarrow f > \frac{5}{27}$</p>	<p>M1 M1 A1 M1 A1 M1 A1 M1 A1 M1 M1 A1</p> <p style="text-align: right;">12</p>
<p>7 a</p> <p>Using conservation of momentum and law of restitution: <i>AB</i> collision $2m \times 2u = 2mv_A + mv_B \Rightarrow 4u = 2v_A + v_B$ $0.5(2u) = v_B - v_A \Rightarrow v_B = u + v_A$ $4u = 2v_A + u + v_A$ $v_A = u, v_B = 2u$ <i>BC</i> collision $m \times 2u - m \times 2u = mv_C + mv_B \Rightarrow v_C = -v_B$ $0.5(2u - (-2u)) = v_C - v_B \Rightarrow v_C = 2u + v_B$ $2u + v_B = -v_B$ $v_B = -u, v_C = u$ <i>BA</i> collision $2mu - mu = 2mv_A + mv_B \Rightarrow 2v_A + v_B = u$ $0.5(u - (-u)) = v_B - v_A \Rightarrow v_B = u + v_A$ $2v_A + u + v_A = u$ $v_A = 0, v_B = u$ so no further collisions</p>	<p>M1 A1 (or equiv) M1 A1 (or equiv) M1 A1 A1 M1 (repetition) A1 A1 M1 (repetition) A1 A1</p>

b	<p>Initial KE: $\frac{1}{2}(2m)(2u)^2 + \frac{1}{2}m(2u)^2 = 6mu^2$</p> <p>Final KE: $\frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2$</p> <p>Loss in KE = $5mu^2$</p>	<p>M1 A1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">17</p>
<p>8 a</p> <p>b</p> <p>c</p> <p>d</p>	<p>Using conservation of momentum and law of restitution:</p> <p>$mu = mv_B + 2mv_C \Rightarrow u = v_B + 2v_C$</p> <p>$ue = v_C - v_B \Rightarrow v_B = v_C - ue$</p> <p>$u = v_C - ue + 2v_C \Rightarrow v_C = \frac{u(1+e)}{3}, v_B = \frac{u(1+e)}{3} - ue = \frac{u(1-2e)}{3}$</p> <p>$v_B < 0$ so $1 - 2e < 0, e > \frac{1}{2}$</p> <p>$v_B = -0.2u, v_C = 0.6u$</p> <p>$-0.2mu = mv_B + 2mv_A \Rightarrow -0.2u = v_B + 2v_A$</p> <p>$0.8 \times 0.2u = v_B - v_A \Rightarrow v_B = v_A + 0.16u$</p> <p>$-0.2u = v_A + 0.16u + 2v_A$</p> <p>$v_A = -0.12u, v_B = 0.04u < v_C$ so no further collisions</p> <p>KE before: $\frac{1}{2}mu^2$</p> <p>KE after: $\frac{1}{2}(2m)(0.12u)^2 + \frac{1}{2}m(0.04u)^2 + \frac{1}{2}(2m)(0.6u)^2 = 0.3752mu^2$</p> <p>KE lost: $0.1248mu^2$</p>	<p>M1 A1 (or equiv)</p> <p>M1 A1 (or equiv)</p> <p>M1 A1 A1</p> <p>M1 A1</p> <p>B1 B1</p> <p>M1</p> <p>M1</p> <p>A1 A1</p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">19</p>

Statics of rigid bodies

1. A uniform ladder AB , of mass 20 kg and length 4 m , has end A on rough horizontal ground. The coefficient of friction between the ladder and the ground is 0.7 . The other end B of the ladder rests against a smooth vertical wall. The ladder rests in equilibrium in a vertical plane perpendicular to the wall, and makes an angle of 40° with the wall. A man of mass 100 kg climbs a distance $x\text{ m}$ up the ladder, at which stage the ladder is on the point of slipping.

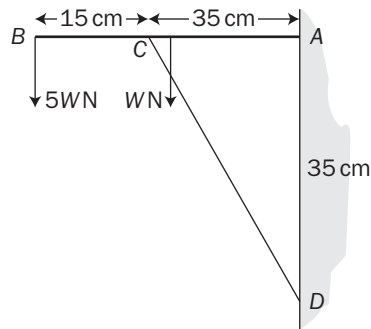
(a) Find the value of x .

The man has an assistant, of mass m . The assistant stands on the ladder at A , which enables the man to climb to the top without the ladder slipping.

(b) Find the minimum value of m for which this is possible.

(c) Explain implications of modelling the man as a particle.

2.



The diagram shows a bracket for suspending a hanging basket of flowers.

It comprises a uniform rod AB , of length 50 cm and weight WN , which is freely hinged to a wall at A . A basket of flowers of weight $5WN$ is suspended from B .

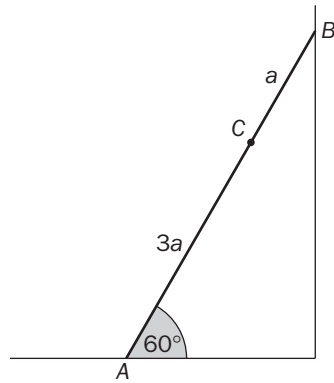
The bracket is supported by a light rod CD , attached to the rod at C , where $AC = 35\text{ cm}$, and to the wall at D , which is 35 cm vertically below A .

Giving exact answers, find

(a) the thrust in the rod CD

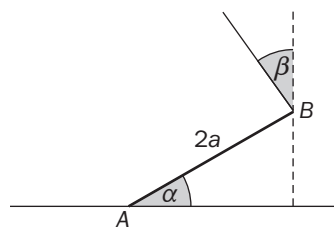
(b) the horizontal and vertical components of the reaction at the hinge A .

3.



The diagram shows a uniform rod, AB , of length $4a$ and mass m , resting against a smooth vertical wall. The end A is on rough horizontal ground. The rod makes an angle of 60° with the horizontal. A particle of mass m is attached to the rod at C , where $AC = 3a$. The rod is in limiting equilibrium. Calculate the coefficient of friction between the rod and the ground.

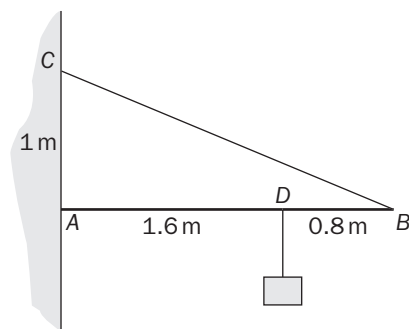
4.



A uniform rod AB has weight W and length $2a$. A light rope is attached to end B . The rope lifts B off the ground. The end A stays on the ground, which is rough and horizontal. The rod is in limiting equilibrium and makes an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The rope makes an angle β to the vertical, as shown in the diagram. The coefficient of friction between the rod and the ground is 0.5.

- Show that the normal reaction on the rod at A is $\frac{4}{5}W$.
- Find the value of β .
- Find the tension in the rope.

5.



The diagram shows a uniform girder AB , of length 2.4 m and mass 80 kg. It is hinged to a vertical wall at A and is held in a horizontal position by means of a light cable BC attached to the wall at C , which is 1 m vertically above A . A load of mass 120 kg is suspended from the girder at D , where $AD = 1.6$ m.

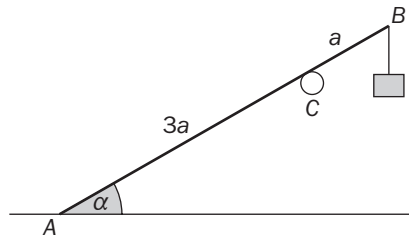
- Find the tension in the cable.
- Find the magnitude and direction of the reaction at the hinge A .

6. A uniform ladder AB , of mass m and length $2a$, has one end A on rough horizontal ground and the other end B against a smooth vertical wall. The coefficient of friction between the ladder and the ground is $\frac{2}{3}$.

A load of mass $5m$ is attached to the ladder at B . The ladder is prevented from slipping by a horizontal force of magnitude P , which is applied at A in a direction perpendicular to the wall. The ladder makes an angle α with the horizontal, where $\tan \alpha = \frac{4}{3}$.

- (a) Find the reaction of the wall on the ladder.
 (b) Find, in terms of m and g , the range of values of P for which the ladder remains in equilibrium.

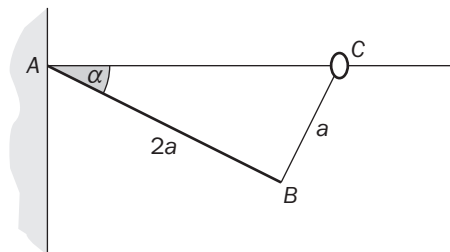
7.



The diagram shows a uniform rod AB , of length $4a$ and mass $5m$, which rests with end A on rough horizontal ground. The rod rests against a smooth peg at C , where $AC = 3a$. A load of mass m is suspended from the rod at B . The coefficient of friction between the rod and the ground is μ . The rod makes an angle α with the horizontal, where $\tan \alpha = \frac{1}{2}$.

- (a) Find the reaction force acting on the rod at C .
 (b) If the rod is in limiting equilibrium, find the value of μ .

8.

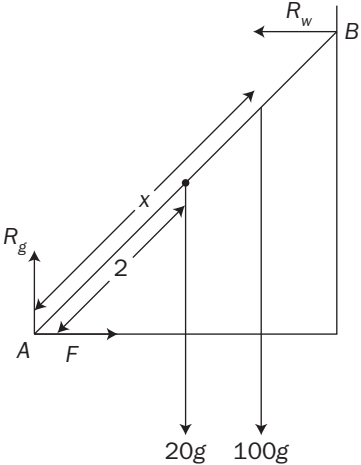


The diagram shows a heavy uniform rod AB of length $2a$ and mass $4m$. It is freely hinged to a vertical wall at A , and the end B is supported by a light string of length a which is attached to a small ring C , of mass m . A light rough horizontal rod projects from the wall at A , and the ring slides on this rod. The coefficient of friction between the ring and the horizontal rod is μ .

When the angle $\widehat{ABC} = 90^\circ$ the system is in limiting equilibrium.

- (a) Calculate the tension in the string BC .
 (b) Calculate the value of μ .
 (c) Find the horizontal and vertical components of the reaction at the hinge A .

Statics of rigid bodies

Question Number	Solution	Marks
<p>1 a</p>	 <p>Resolving vertically: $R_g = 120g = 1176$ $F = \mu R_g = 0.7 \times 1176 = 823.2 \text{ N} = R_w$</p> <p>Taking moments about A: $R_w \times 4 \sin 50^\circ = 20g \times 2 \cos 50^\circ + 100g \times x \cos 50^\circ$ $x = 3.6 \text{ m}$</p>	<p>M1 A1 M1 A1</p> <p>M1 A1</p>
<p>b</p>	<p>$R_g = 120g + mg, F = 0.7(120g + mg) = R_w$ $0.7(120g + mg) \times 4 \sin 50^\circ = 20g \times 2 \cos 50^\circ + 100g \times 4 \cos 50^\circ$ $m = 11.86 \text{ kg}$</p>	<p>M1 A1 M1 A1</p>
<p>c</p>	<p>The mass acts <i>on</i> the ladder at a single point.</p>	<p>E1</p>
<p>2 a</p>	<p>Taking moments about A and the reaction perpendicular to the wall at D to be X: $0.35X = 0.25W + 0.5 \times 5W$ $X = \frac{55}{7}W = T \sin 45^\circ$ $T = \frac{55\sqrt{2}}{7}W$</p>	<p>M1 A1 M1 A1</p>
<p>b</p>	<p>Y, the vertical force acting at D, is the same as X by symmetry $R_v = 6W - Y = -\frac{13}{7}W$ so $\frac{13}{7}W$ down $R_h = X = \frac{55}{7}W$ right</p>	<p>B1 M1 A1 B1</p>
		<p>11</p>
		<p>8</p>

3	<p>Taking moments about A:</p> $R_w \sin 60^\circ \times 4a = mg \cos 60^\circ \times 2a + mg \cos 60^\circ \times 3a$ $R_w = \frac{5}{4\sqrt{3}} mg = F$ $R_g = 2mg$ $\mu = \frac{F}{R} = \frac{5}{8\sqrt{3}} = 0.361$	<p>M1 A1 B1 M1 A1</p> <p style="text-align: right;">5</p>
4 a	<p>$F = \mu R = 0.5R$ $\sin \alpha = \frac{3}{5}, \cos \alpha = \frac{4}{5}$ Taking moments about B: $W \times \frac{4}{5}a + \frac{1}{2}R \times \frac{3}{5}(2a) = R \times \frac{4}{5}(2a)$ $R = \frac{4}{5}W$</p> <p>b $F = 0.5R = T \sin \beta, T \cos \beta = W - R$ $\Rightarrow \tan \beta = \frac{0.5R}{W - R} \Rightarrow \beta = 63.4^\circ$</p> <p>c It follows from substituting back that $T = 0.447WN \left(\text{or } \frac{W}{\sqrt{5}} \right)$</p>	<p>B1 B1 (both soi)</p> <p>M1 A1</p> <p>M1 M1 A1</p> <p>M1 A1</p> <p style="text-align: right;">9</p>
5 a	<p>Taking moments about A and the reaction perpendicular to the wall at C to be X: $X = 80g \times 1.2 + 120g \times 1.6 = 2822.4 \text{ N}$ Cable length is 2.6m (Pythagoras) At C: $X = T \cos \alpha \Rightarrow T = 2822.4 \div \frac{2.4}{2.6} = 3057.6 \text{ N}$</p> <p>b At A, reaction perpendicular to the wall is $X = 2822.4 \text{ N}$ Taking reaction parallel to the wall at A to be Y and parallel to the wall at C to be Z: $Y = 80g + 120g - Z$ $Z = T \sin \alpha = 3057.6 \times \frac{1}{2.6} = 1176 \text{ N}$ $Y = 200g - 1176 = 784 \text{ N}$ $\sqrt{2822.4^2 + 784^2} = 2929.3 \text{ N}$ $\tan^{-1} \left(\frac{784}{2822.4} \right) = 15.5^\circ$ up and to the right</p>	<p>M1 A1 B1 M1 A1</p> <p>B1</p> <p>M1 M1 A1 A1 M1 A1 M1 A1</p> <p style="text-align: right;">14</p>

<p>6 a</p>	<p>Taking moments about A: $mga \cos \alpha + 5mg \cos \alpha \times 2a = R_w \times 2a \sin \alpha$ $\sin \alpha = \frac{4}{5}, \cos \alpha = \frac{3}{5}$ $\Rightarrow R_w = \frac{33}{8} mg$</p> <p>b</p> <p>$R_g = 6mg, F = \mu R_g = \frac{2}{3} \times 6mg = 4mg$ Minimum value: $P = R_w - F = \frac{1}{8} mg$ Maximum value: $P = R_w + F = \frac{65}{8} mg$</p>	<p>M1 B1 A1 M1 A1 M1 A1 M1 A1</p> <p style="text-align: right;">9</p>
<p>7 a</p>	<p>Taking moments about A: $\sin \alpha = \frac{1}{\sqrt{5}}, \cos \alpha = \frac{2}{\sqrt{5}}$ $5mg \times 2a \cos \alpha + mg \times 4a \cos \alpha = R_C \times 3a$ $R_C = \frac{28mg}{3\sqrt{5}} = 40.9mN$</p> <p>b</p> <p>$R_C \sin \alpha = F = \frac{28}{15} mg$ $R_A = 6mg - R_C \cos \alpha = \frac{34}{15} mg$ $\mu = \frac{F}{R} = \frac{14}{17}$</p>	<p>B1 M1 A1 M1 A1 M1 A1 M1 A1</p> <p style="text-align: right;">9</p>
<p>8 a</p>	<p>Take moments about A for the rod: $T \times 2a = 4mg \times a \cos \alpha$ Therefore $2T = 4mg \times \frac{2}{\sqrt{5}}$ Hence $T = \frac{4mg}{\sqrt{5}} = \frac{4\sqrt{5}}{5} mg$</p> <p>b</p> <p>Resolve vertically for the ring: $R_c = T \cos \alpha + mg$ $= \frac{4}{5} \sqrt{5} mg \times \frac{2}{\sqrt{5}} + mg$ $= \frac{13}{5} mg$</p> <p>Resolve horizontally for the ring: $T \sin \alpha = \mu R_c$ Therefore $\mu \times \frac{13mg}{5} = \frac{4\sqrt{5}}{5} mg \times \frac{1}{\sqrt{5}}$ so $\mu = \frac{4mg}{13mg} = \frac{4}{13}$</p>	