1 **a** Prove by induction that for all positive integers *n*, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$

(5 marks)

b Hence deduce an expression, in terms of *n*, for $\sum_{r=1}^{2n-1} r(r+1)$ in the form $an(bn^2 - c)$ where *a*, *b* and *c* are rational numbers to be found. **(2 marks)**

2 Prove by induction that for all positive integer *n*, $\sum_{r=1}^{n} \frac{r-1}{r!} = \frac{n!-1}{n!}$ **(6 marks)**

3 Prove by induction that for all positive integers *n*, $5^{2n} + 11$ is divisible by 6 **(7 marks)**

4 Prove by induction that for all positive integers *n*, $11^n - 7^n$ is divisible by 4 **(7 marks)**

5 Prove by induction that for all positive integers *n*, $n^3 + 9n^2 + 5n$ is divisible by 3 **(7 marks)**

6 Prove by inducution that for all positive integers *n*, $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} n+1 & -n \\ n & -(n-1) \end{pmatrix}$ **(6 marks)**

7 **a** Prove by induction that for all positive integers *n*, $\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ \frac{1}{2}(3^n - 1) & 1 \end{pmatrix}$ **(7 marks)**

b Given that matrix M = $\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$

Hence find an expression for $(\mathbf{M}^n)^{-1}$ in terms of *n* **(3 marks)**