

Do not write on these sheets.

For **Pearson Edexcel**
Level 3 GCE

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Time: 2 hours

Churchill Paper 1A

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Instructions

- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.



Written by Shaun Armstrong

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Answer ALL questions.

1 The straight line L_1 has the equation $y = \frac{1}{3}x + 4$

The straight line L_2 passes through the point $M(p - 3, 10)$ and the point $N(p, p - 1)$, where p is a constant.

Given that L_1 and L_2 are perpendicular, find the value of p . (4)

2 The equation $ax^2 - 7x + 3 = 0$ has no real roots.

Find the smallest possible integer value of the constant a . (3)

3 (a) Write down the angle between the vector $\mathbf{i} + \mathbf{j}$ and the vector $\mathbf{i} - \mathbf{j}$. (1)

(b) Calculate the angle between the vector $2\mathbf{i} + 5\mathbf{j}$ and the vector $4\mathbf{i}$. (2)

(c) Two forces, $\mathbf{A} = (14\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{B} = (2\mathbf{i} - 6\mathbf{j})$ N, act on a ball.
Show that the magnitude of \mathbf{A} is \sqrt{k} times larger than the magnitude of \mathbf{B} , where k is an integer. (3)

4 The curve C has the equation $y = 2x^2 - 3x + k$, where k is a constant.

The minimum value of C is $\frac{5}{2}$.

(a) Explain how you know that C does not cross the x -axis. (1)

(b) Find the coordinates of the point where C crosses the y -axis. (4)

5 A circle has the equation $x^2 + y^2 + 2x - 8y + c = 0$, where c is a constant.

AB is a diameter of the circle.

The point A has coordinates $(-7, 2)$.

(a) Find the value of c . (2)

(b) Find the radius of the circle in the form $a\sqrt{10}$, where a is an integer. (3)

The point C lies on the circle such that $AC = 12$.

(c) Find the length BC . (3)

6 (a) Solve the equation

$$\log_4(x - 4) = 2$$
(2)

(b) Evaluate

$$2 \log_p \left(\frac{3}{p} \right) + \log_p \left(\frac{p^7}{9} \right)$$
(3)

7 $f(x) = 7 - 2x(x - 1)^2$

Find the set of values of x for which $f(x)$ is increasing. (5)

8 $f(x) = x^3 + ax^2 - 21x + b$

where a and b are constants.

Given that $f(x)$ can be expressed in the form $(x - 1)(x - 4)(x + c)$, find the value of the constant c . (4)

9 (a) “For any positive integer n , the value of $3n^2 + n - 1$ is a prime number.”

Find a counter example to disprove the statement above.

(2)

(b) Given that n is a positive integer, prove that the value of $3n^2 + n$ is always even.

(3)

10 A student was asked to find all the solutions in the interval $0^\circ \leq x \leq 360^\circ$ to the equation

$$\sin^2 x - 3 \sin x \cos x = 0$$

Here is the student's answer:

$$\sin^2 x - 3 \sin x \cos x = 0$$

$$\sin x (\sin x - 3 \cos x) = 0$$

$$\text{Either } \sin x = 0 \text{ or } \sin x - 3 \cos x = 0$$

$$\text{If } \sin x = 0 \text{ then } x = 0^\circ \text{ or } 180^\circ$$

$$\text{If } \sin x - 3 \cos x = 0$$

$$\sin x = 3 \cos x$$

$$\frac{1}{3} = \frac{\cos x}{\sin x}$$

$$\tan x = \frac{1}{3}$$

$$\text{Giving } x = 18.4^\circ \text{ or } 198.4^\circ \text{ (1dp)}$$

$$\text{Therefore } x = 0^\circ, 18.4^\circ \text{ (1dp), } 180^\circ, 198.4^\circ \text{ (1dp)}$$

(a) Identify the two errors made by the student.

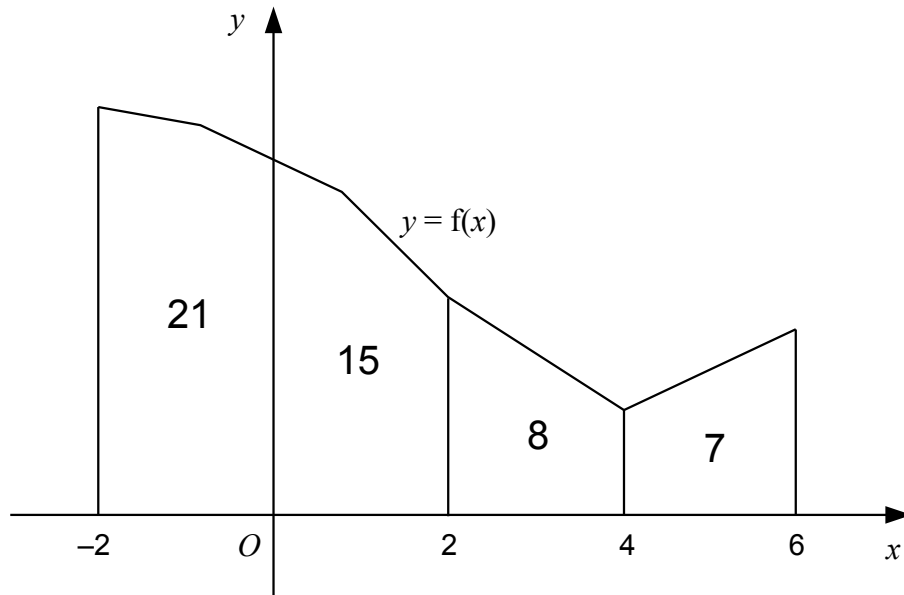
(2)

(b) Find all the correct solutions to the equation.

Give your answers correct to 1 decimal place where appropriate.

(3)

- 11 A sketch of the graph of $y = f(x)$ is shown for $-2 \leq x \leq 6$



Four regions are bounded by $y = f(x)$, the x -axis and the lines $x = -2$, $x = 0$, $x = 2$, $x = 4$ and $x = 6$. The areas of these regions are 21, 15, 8 and 7 as shown on the diagram.

- (a) Write down the value of $\int_0^4 f(x) \, dx$ (1)
- (b) Explaining your reasoning, evaluate $\int_2^4 f(x-2) \, dx$ (2)
- (c) Explain why $f(0) < 10.5$ (2)
- (d) Explaining your reasoning, evaluate $f(2) - f(6)$ (3)

- 12 Differentiate $f(x) = 2x^3$ from first principles. (4)

- 13** A new housing development is expected to cause a decline in the population of wood mice in the area.

A model is proposed for the number of mice, N , to be given by

$$N = 250 - 50\sqrt{t}$$

where t is the number of years after the development begins, at the start of 2005.

(a) Using this model,

(i) state the number of mice at the start of 2005, (1)

(ii) find the year in which the number of mice first fell below 80, (2)

(iii) find the rate at which the number of mice was decreasing at the start of 2014. (2)

Another model is proposed in which

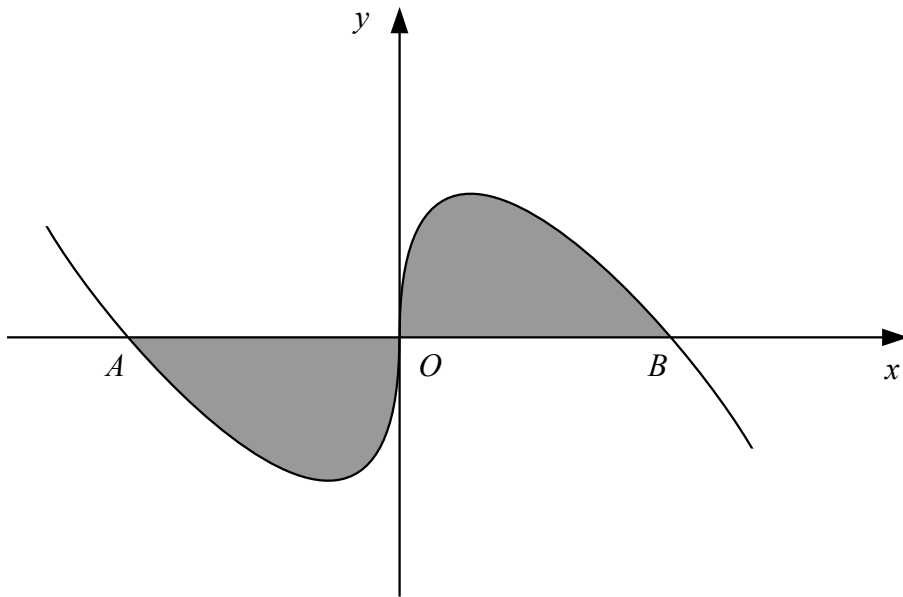
$$N = ae^{-bt}$$

where a and b are positive constants and t is the number of years after the start of 2005.

Both models give the same number of mice for $t = 0$ and $t = 4$.

(b) Find the value of a and the value of b . (3)

14



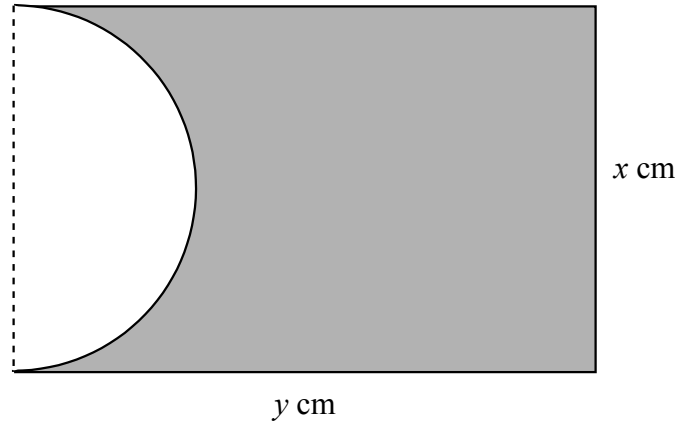
A sketch of the curve with equation $y = 4x^{\frac{1}{3}} - x$ is shown above.

The curve crosses the x -axis at the point $A(-8, 0)$, the origin O , and the point B .

- (a) State the coordinates of the point B and explain how you know. (1)
- (b) Find $\int (4x^{\frac{1}{3}} - x) dx$ (3)
- (c) Find the total area of the shaded regions enclosed by the curve and the x -axis. (3)

- 15 (a) Find the first three terms, in descending powers of x , of the expansion of $\left(4 - \frac{1}{x}\right)^7$ (3)
- (b) Find the value of x for which your expansion can be used to estimate the value of 3.95^7 (1)
- (c) Use your answer to part (b) to estimate the value of 3.95^7
Find out the number of significant figures to which your estimate is accurate. (2)

16



The diagram shows the design for a tool to be used to scrape oven doors. The tool is made from a rectangular sheet of metal measuring x cm by y cm. A semicircle of diameter x cm is removed from the metal sheet as shown.

Given that the original rectangular sheet of metal must have an area of 100 cm^2 ,

(a) show that the perimeter, P cm, of the tool is given by

$$P = \frac{200}{x} + x \left(1 + \frac{1}{2} \pi \right) \quad (3)$$

The edges of the tool must be treated.

This means that the cost of making the tool can be reduced by minimising its perimeter.

(b) Find the minimum value of P and show that your value is a minimum. (5)

17 (a) Solve the equation $12x^2 + 5x - 3 = 0$ (3)

(b) Hence, or otherwise,

(i) find all solutions in the interval $0^\circ \leq A < 360^\circ$ of the equation

$$12 \sin^2 A - 5 \cos A = 9 \quad (4)$$

(ii) find all solutions in the interval $0^\circ \leq B < 90^\circ$ of the equation

$$12 \sin^2 3B - 5 \cos 3B = 9 \quad (2)$$

TOTAL FOR PAPER IS 100 MARKS