Further Maths AS Summer Work

Core 1H

EdExcel Further Pure 1

Complex Numbers

Section 1: Introduction to complex numbers

Exercise

1. Find the roots of the following equations:
   (i) $z^2 + 25 = 0$
   (ii) $4z^2 + 9 = 0$
   (iii) $z^2 - 2z + 2 = 0$
   (iv) $4z^2 + 4z + 5 = 0$

   (10 marks)

2. In each of the following cases find
   (a) $z_1 + z_2$ (b) $z_1 - z_2$ (c) $z_1 z_2$ (d) z_1^*
   (e) z_2^* (f) $z_1^* + z_2^*$ (g) $z_1^* - z_2^*$ (h) $z_1^* z_2^*$

   (i) $z_1 = 2 + 3i$; $z_2 = 1 - 2i$
   (ii) $z_1 = -2i$; $z_2 = 3 + i$
   What do you notice about the results?

   (20 marks)

3. Given that $z = (a + i)^4$ where a is real, find values for a such that
   (a) z is real,
   (b) z is wholly imaginary.

   (8 marks)

4. Simplify and write in the form $a + bi$
   (i) $\dfrac{1}{3+2i} + \dfrac{1}{3-2i}$
   (ii) $3 + i + \dfrac{4}{3-i}$
   (iii) $\dfrac{3}{1-i} - \dfrac{2i}{2+i}$

   (8 marks)

5. Find the values of p and q given that one root of the equation $z^2 + pz + q = 0$ is
   (i) $2 - i$
   (ii) $1 - 3i$
   (iii) $2i$
   (iv) $5 - 3i$

   (8 marks)

6. Given that $\dfrac{5}{a+bi} + \dfrac{2}{1+3i} = 1$, where a and b are real, find a and b.

   (6 marks)

Total : 60

Complex Numbers – Exam Questions

1 The complex numbers $2 + 3i$ and $4 - i$ are denoted by z and w respectively. Express each of the following in the form $x + iy$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w, where z^* is the complex conjugate of z, [3]

(iii) $\frac{1}{w}$. [2]

(Q3, June 2005)

2 Use an algebraic method to find the square roots of the complex number $21 - 20i$. [6]

(Q4, June 2005)

3 **(i)** Express $(1 + 8i)(2 - i)$ in the form $x + iy$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\frac{1 + 8i}{2 + i}$ in the form $x + iy$. [3]

(Q1, Jan 2006)

4 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3i$.

(i) Write down the other root. [1]

(Q3, June 2006)

5 The complex number z satisfies the equation $z + 2iz^* = 12 + 9i$. Find z, giving your answer in the form $x + iy$. **[5]**

(Q3, Jan 2010)

6 The complex numbers $3 - 2i$ and $2 + i$ are denoted by z and w respectively. Find, giving your answers in the form $x + iy$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(iz)^2$, [3]

(iii) $\frac{z}{w}$. [3]

(Q5, June 2006)

7 Express $\frac{2 + 3i}{5 - i}$ in the form $x + iy$, showing clearly how you obtain your answer. [4]

(Q1, Jan 2009)

8 Use an algebraic method to find the square roots of the complex number $15 + 8i$. [6]

(Q2, Jan 2007)

9 **(i)** Use an algebraic method to find the square roots of the complex number $5 + 12i$. [5]

(ii) Find $(3 - 2i)^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

(Q9, June 2008)