

## Mechanics 8 – Kinematics 1: Rates of Change

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. For each of the following functions, find  $f'(x)$ .

(i)  $f(x) = 2x + 1$

(ii)  $f(x) = x^3 - 5x$

(iii)  $f(x) = \frac{1}{x^3}$

(iv)  $f(x) = \sqrt[3]{x}$

(v)  $f(x) = \frac{2}{x} - \frac{3}{x^2}$

(vi)  $f(x) = 4\sqrt{x} - \frac{3}{\sqrt{x}}$

(vii)  $f(x) = 3x^{-5} - 2x^{-7}$

(viii)  $f(x) = 2x^{\frac{2}{3}} - 5x^{-\frac{2}{3}}$

(ix)  $f(x) = 3x^4 - 4x^{\frac{3}{2}} + \frac{1}{x}$

(x)  $f(x) = x(x + 2)$

2. Given that  $y = 12x - x^3$ ,

(i) Find the gradient of the curve at the origin.

(ii) Find the coordinates of the two points where the gradient is zero.

Section 2 – Consolidation of this week's topic. Please complete all questions.

- The displacement of a particle  $P$  from a fixed point  $O$  at time  $t$  seconds is given by  $s = t^3 + 2t^2 + 3t + 4$ . Find expressions for the velocity  $v$   $\text{ms}^{-1}$  and the acceleration  $a$   $\text{ms}^{-2}$  and hence find the values of  $v$  and  $a$  when  $t = 2$ . **(5 marks)**
- The displacement,  $s$  m, after time  $t$  seconds, of a particle from a fixed point (the origin) is given by  $s = 2t^2 + 3t - 2$ .
  - Find the initial displacement and the initial velocity.
  - Find any times at which the velocity is zero
  - Find any times at which the particle is at the origin. **(3+3+5=11 marks)**
- Particle  $P$  has a displacement,  $s$  metres, from a fixed point  $O$  given by  $s = 6t^2 - t^3$ , where  $t$  is the time in seconds.
  - Sketch the displacement-time and velocity-time graphs for the time interval  $0 \leq t \leq 6$ .
  - Find the times when the particle is at  $O$ .
  - Find the greatest displacement from  $O$  within the time interval  $0 \leq t \leq 6$ .
  - Find the greatest speed attained in this same time interval. **(6+2+5+5=18 marks)**

4. Given that  $s$  is measured in metres and  $t$  in seconds, find the maximum (positive) velocity of an object whose displacement is given by:

(a)  $s = 15t + 6t^2 - t^3$

(b)  $s = \frac{-t^5}{20} + \frac{t^4}{4} - \frac{t^3}{3} + 11t$

(5+5=10 marks)

5. A computer is used to track the motion of a lizard, running back and forth along a track. The computer logs its position over a 7 second interval and computes that its displacement can be approximated by the following quartic function:

$$s = \frac{1}{6}t^4 - 2t^3 + 5t^2 + 2t \quad \text{for } 0 \leq t \leq 7$$

where  $s$  is measured in metres and  $t$  in seconds. Calculate the lizard's greatest **speed** over these seven seconds.

(6 marks)

(Total 50 Marks)

Section 3 – Extension question. If you are aiming for a top grade, you should attempt these questions.

Find the maximum displacement during the first second of motion, in metres, of an object whose acceleration is given by  $a = (6t - 4)\text{ms}^{-2}$ , with the conditions that  $s = 0$  at both  $t = 0$  and  $t = 1$ .