

Mechanics 09 – Kinematics 2: Integration

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please **complete** all questions.

1. Evaluate the following definite integrals.

$$\begin{array}{lll}
 \text{(i)} \int_{-1}^1 (4x+5)dx & \text{(ii)} \int_{-1}^0 (6x^2 - 2x)dx & \text{(iii)} \int_2^4 (x^2 - x + 3)dx \\
 \text{(iv)} \int_{-1}^2 (2+x-x^2)dx & \text{(v)} \int_{-1}^2 (x^3 - x + 4)dx & \text{(vi)} \int_1^3 \frac{1}{x^3} dx \\
 \text{(vii)} \int_1^9 \frac{1}{\sqrt{x}} dx & \text{(viii)} \int_1^4 \frac{2-x+3x^2}{\sqrt{x}} dx & \text{(ix)} \int_1^2 \frac{x^2-1}{x^4} dx
 \end{array}$$

Section 2 – Consolidation of this week's topic. Please **complete** all questions.

- A particle is moving along the x -axis. At time t seconds (where $t \geq 0$), the velocity of P is v ms^{-1} in the direction of x increasing, where $v = 12 - t - t^2$. Find the acceleration of P when P is instantaneously at rest. **(6 marks)**
- At time t seconds, where $t \geq 0$, the velocity v ms^{-1} of a particle moving in a straight line is given by $v = 12 + t - 6t^2$. When $t = 0$, P is at a point O on the line. Find
 - The magnitude of the acceleration of P when $v = 0$
 - The distance of P from O when $v = 0$**(6+6 = 12 marks)**
- A particle P is moving on the x -axis. At time t seconds, the velocity of P is $(4t-t^2)$ ms^{-1} in the direction of x increasing. At time $t = 0$, P is at the origin O . Find
 - The value of x when $t > 0$ and P is at rest
 - The total distance moved by P in the interval $0 \leq t \leq 5$.**(5+5 = 10 marks)**
- A particle P moves in a straight line so that, at time t seconds, its velocity v ms^{-1} is given by

$$v = \begin{cases} 4, & 0 \leq t \leq 2 \\ 5 - \frac{4}{t^2}, & t > 2 \end{cases}$$
 - Sketch a velocity-time graph to illustrate the motion of P .
 - Find the distance moved by P in the interval $0 < t \leq 5$.**(3+5 = 8 marks)**
- A particle P moves in a straight line so that, at time t seconds, its acceleration a ms^{-2} is given by

$$a = \begin{cases} 6t - t^2, & 0 \leq t \leq 2 \\ 8 - t, & t > 2 \end{cases}$$
 When $t = 0$ the particle is at a fixed point O on the line. Find
 - The speed of P when $t = 2$,
 - The speed of P when $t = 4$,
 - The distance from O to P when $t = 4$.**(5+5+7 = 17 marks)**

(Total 55 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

A particle P moves along the x -axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1} , where

$$v = \begin{cases} 10t - 2t^2, & 0 \leq t \leq 6, \\ \frac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O . Find the displacement of P from O when

- (a) $t = 6$,
- (b) $t = 10$.