Godalming College

Mechanics 8 – Kinematics 1 - Solutions

Section 1

1. (i) $f'(x) = 2$ (ii) $f'(x) = 3x^2-5$ (iii) $f'(x) = -3x^{-4}$ (iv) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

   (v) $f'(x) = -2x^{-2}+6x^{-3}$ (vi) $f'(x) = 2x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$ (vii) $f'(x) = -15x^{-6}+14x^{-8}$

   (viii) $f'(x) = \frac{4}{3}x^{-\frac{1}{3}} + \frac{10}{3}x^{-\frac{5}{3}}$ (ix) $f'(x) = 12x^3 - 6x^{\frac{1}{2}} - x^{-2}$ (x) $f'(x) = 2x + 2$

2. (i) $\frac{dy}{dx} = 12 - 3x^2 \Rightarrow$ at $x = 0$ $\frac{dy}{dx} = 12$

   (ii) $\frac{dy}{dx} = 12 - 3x^2 = 0 \Rightarrow x = 2$ or $-2 \Rightarrow$ coordinates are (2, 16) and (-2, -16)

Section 2

1. $s = t^3 + 2t^2 + 3t + 4$

$$v = \frac{ds}{dt} = 3t^2 + 4t + 3 \quad \checkmark \checkmark$$

$$a = \frac{dv}{dt} = 6t + 4 \quad \checkmark$$

When $t = 2$, $v = 3 \times 2^2 + 4 \times 2 + 3 = 23$ ✓

$a = 6 \times 2 + 4 = 16$ ✓ **(5 marks)**

2. (i) When $t = 0$, $s = -2$, so the initial displacement $= -2$ m. ✓

$s = 2t^2 + 3t - 2$

$$v = \frac{ds}{dt} = 4t + 3 \quad \checkmark$$

When $t = 0$, $v = 3$ so the initial velocity is 3 ms^{-1}. ✓

(ii) $v = 0 \Rightarrow 4t + 3 = 0$ ✓ $\Rightarrow t = -0.75$ ✓

Since this is negative, there are no times for which the velocity is zero. ✓

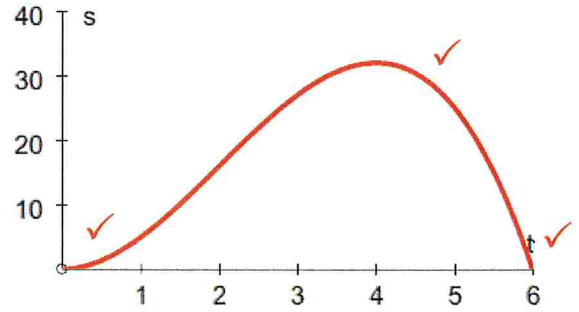
(iii) When $s = 0$, $2t^2 + 3t - 2 = 0$ ✓

$(2t - 1)(t + 2) = 0$ ✓

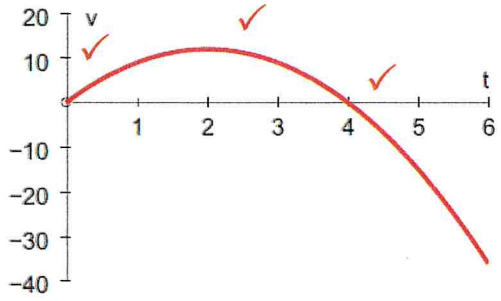
$t = \frac{1}{2}$ ✓ or -2 ✓

The particle is at the origin when $t = \frac{1}{2}$. ✓ **(11 marks)**

3. (i) $s = 6t^2 - t^3 = t^2(6-t)$



$$v = \frac{ds}{dt} = 12t - 3t^2 = 3t(4-t)$$



(ii) The particle is at O when $t = 0$ and when $t = 6$.

(iii) The greatest displacement is when the velocity is zero.

$3t(4-t) = 0$

$t = 0$ or $t = 4$

From the graph, the greatest displacement is when $t = 4$

$s = 4^2(6-4) = 32$

The greatest displacement is 32 m.

(iv) The greatest positive speed is when the acceleration is zero

$$a = \frac{dv}{dt} = 12 - 6t$$

When $a = 0$, $t = 2$

When $t = 2$, $v = 3 \times 2(4-2) = 12$

The greatest negative speed in the time interval is when $t = 6$ (from the graph).

When $t = 6$, $v = 3 \times 6(4-6) = -36$

So the greatest speed in the time interval is 36 ms^{-1}.

(18 marks)

4. Maximum velocity occurs when acceleration = 0

(a) $v = \frac{ds}{dt} = 15 + 12t - 3t^2$ ✓

$a = \frac{dv}{dt} = 12 - 6t = 0$ ✓ $\Rightarrow t = 2$ ✓

$\frac{d^2v}{dt^2} < 0 \Rightarrow$ maximum ✓

$\Rightarrow$ maximum velocity $= 15 + 12 \times 2 - 3 \times 2^2 = 27 ms^{-1}$. ✓

(b) $v = \frac{ds}{dt} = \frac{-t^4}{4} + t^3 - t^2 + 11$ ✓

$a = \frac{dv}{dt} = -t^3 + 3t^2 - 2t = 0$ ✓ $\Rightarrow -t(t-1)(t-2) = 0$ ✓

$\Rightarrow t = 0, 1 \text{ or } 2$ ✓

$\frac{d^2v}{dt^2} < 0$ when $t = 0$ or 2 $\Rightarrow$ maximum

$\Rightarrow$ maximum velocity $= 11\ ms^{-1}$ ✓ (10 marks)

5. $v = \frac{ds}{dt} = \frac{2}{3}t^3 - 6t^2 + 10t + 2$ ✓

$a = \frac{dv}{dt} = 2t^2 - 12t + 10 = 0$ ✓ $\Rightarrow 2(t-1)(t-5) = 0$ ✓

$\Rightarrow t = 1 \text{ or } 5$ ✓

$V_1 = \frac{20}{3}, V_5 = \frac{-44}{3}$ ✓

$\Rightarrow$ maximum speed $= \frac{44}{3}\ ms^{-1}$ ✓ (6 marks)

(Total 50 Marks)

Section 3

$v = 3t^2 - 4t + c \rightarrow s = t^3 - 2t^2 + ct + d$ s = 0, t = 0 so d = 0

s = 0, t = 1 so c = 1

Max s when v = 0 so t = 1/3 or 1 (we know s = 0 when t = 1) so max displacement when t = 1/3, s = 4/27 metres.