Mechanics 9 – Kinematics 2 - Solutions

Section 1

1. (i) 10 (ii) 3 (iii) $\frac{56}{3}$ (iv) $\frac{9}{2}$ (v) $\frac{57}{4}$ (vi) $\frac{4}{9}$

   (vii) 4 (viii) 36.53 (2 d.p.) (ix) $\frac{5}{24}$

Section 2

1. P is at rest when $v = 0$

$$0 = 12 - t - t^2 \checkmark$$

$$t^2 + t - 12 = (t+4)(t-3) = 0 \checkmark$$

$$t = -4, 3 \checkmark \checkmark$$

As $t \geq 0, t = -4$ is rejected. ✓

$$a = \frac{dv}{dt} = -1 - 2t \checkmark$$

When $t = 3$,

$$a = -1 - 2 \times 3 = -7 \checkmark$$

The acceleration of P when P comes to instantaneously to rest is 7 m s^{-2} in the direction of x decreasing. ✓

(8 marks)

2. (a) $a = \frac{dv}{dt} = 1 - 12t$ ✓

When v = 0, $12 + t - 6t^2 = 0 \Rightarrow (4 + 3t)(3 - 2t) = 0$ ✓ ✓

$t \geq 0$ and so $t = \frac{3}{2}$ ✓ $\Rightarrow$ acceleration $= 1 - 12(\frac{3}{2}) = -17$ ✓

So the magnitude of the acceleration is 17 ms^{-2} ✓

(b) $s = \int v.dt = \int 12 + t - t^2.dt = 12t + \frac{1}{2}t^2 - 2t^3 + c$ ✓ ✓

But s = 0 when t = 0 and so c = 0 ✓

So when v = 0, $t = \frac{3}{2}$ ✓

$\Rightarrow s = 12(\frac{3}{2}) + \frac{1}{2}(\frac{3}{2})^2 - 2(\frac{3}{2})^3$ ✓

$\Rightarrow s = 12.375$

= 12.4 m (3sf) ✓

(12 marks)

3. a P is at rest when $v = 0$

$$v = 4t - t^2 = 0$$

$$t(4 - t) = 0$$

As $t > 0$, $t = 4$

$$x = \int v\,dt$$

$$= 2t^2 - \frac{1}{3}t^2 + c$$

When $t = 0, x = 0$

$$0 = 0 - 0 + c = 0 \Rightarrow c = 0$$

$$x = 2t^2 - \frac{1}{3}t^3$$

When $t = 4$

$$x = 2 \times 4^2 - \frac{4^3}{3} = 10\frac{2}{3}$$

b When $t = 5$,

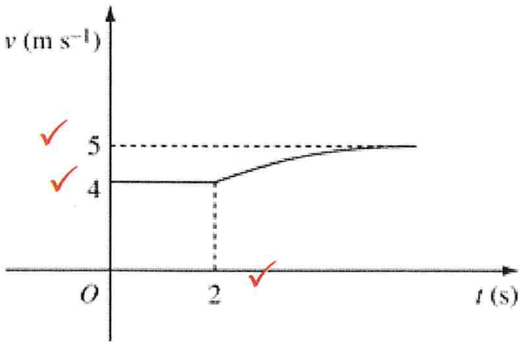
$$x = 2 \times 5^2 - \frac{5^3}{3} = 8\frac{1}{3}$$

In the interval $0 \le t \le 5$, moves to a point $10\frac{2}{3}$ m from O and then returns to a point $8\frac{1}{3}$ m from O.

The total distance moved is $10\frac{2}{3} + \left(10\frac{2}{3} - 8\frac{1}{3}\right) = 13$ m.

(10 marks)

4. a



b In the first two seconds P moves $2 \times 4 = 8$ m

$$s = \int v\,dt = \int (5 - 4t^{-2})\,dt$$

$$= 5t - \frac{4t^{-2}}{-1} + C = 5t + \frac{4}{t} + C$$

When $t = 2, s = 8$

$$8 = 5 \times 2 + \frac{4}{2} + C = 12 + C \Rightarrow C = -4$$

$$s = 5t + \frac{4}{t} - 4$$

When $t = 5$,

$$s = 5 \times 5 + \frac{4}{5} - 4 = 21.8$$

In the interval $0 \le t \le 5$, P moves 21.8 m.

(8 marks)

5. **a** For $0 \le t \le 2$

$$v = \int a \, dt = \int (6t - t^2) dt$$

$$= 3t^2 - \frac{1}{3}t^3 + c, \text{ where } c \text{ is a constant of integration.}$$

When $t = 0, v = 0$

$$0 = 0 - 0 + c \Rightarrow c = 0$$

$$v = 3t^2 - \frac{1}{3}t^3$$

When $t = 2$,

$$v = 3 \times 2^2 - \frac{2^3}{3} = \frac{28}{3}$$

The speed of P when $t = 2$ is $\frac{28}{3}$ m s^{-1}.

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b For $t > 2$,

$$v = \int a \, dt = \int (8 - t) dt$$

$$= 8t - \frac{1}{2}t^2 + k, \text{ where } k \text{ is a constant of integration.}$$

From a, when $t = 2, v = \frac{28}{3}$

$$\frac{28}{3} = 16 - \frac{4}{2} + k \Rightarrow k = -\frac{14}{3}$$

$$v = 8t - \frac{1}{2}t^2 - \frac{14}{3}$$

When $t = 4$,

$$v = 32 - 8 - \frac{14}{3} = \frac{58}{3}$$

The speed of P when $t = 4$ is $\frac{58}{3}$ m s^{-1}.

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c For $0 \le t \le 2$,

$x = \int v\,dt = \int\left(3t^2 - \frac{1}{3}t^3\right)dt = t^3 - \frac{1}{12}t^4 + l$, where l is a constant of integration.

When $t = 0, x = 0$

$0 = 0 - 0 + l \Rightarrow l = 0$ ✓

When $t = 2$,

$x = 2^3 - \frac{2^4}{12} = \frac{20}{3}$ (1) ✓

For $t > 2$,

$x = \int v\,dt = \int\left(8t - \frac{1}{2}t^2 - \frac{14}{3}\right)dt = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + m$,

where m is a constant of integration.

From (1) above

When $t = 2, x = \frac{20}{3}$ ✓

$\frac{20}{3} = 16 - \frac{8}{6} - \frac{28}{3} + m \Rightarrow m = \frac{4}{3}$ ✓

$x = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + \frac{4}{3}$

When $t = 4$.

$x = 64 - \frac{64}{6} - \frac{56}{3} + \frac{4}{3} = 36$

The distance from O to P when $t = 4$ is 36 m. ✓ **(17 marks)**

(Total 55 Marks)

Section 3

(a)	$v = 10t - 2t^2$, $s = \int v\,dt$	M1	
	$= 5t^2 - \frac{2t^3}{3}(+C)$	A1	
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1	(3)
(b)	$\underline{s} = \int v\,dt = \frac{-432t^{-1}}{-1}(+K) = \frac{432}{t}(+K)$	B1	
	$t = 6, s = \text{“36”} \Rightarrow 36 = \frac{432}{6} + K$	M1*	
	$\Rightarrow K = -36$	A1	
	At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1	
		A1	(5) [8]