Pure 11 – Vectors 1

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. In the expansion of $(1+\frac{x}{2})^n$ in ascending powers of x the coefficient of x^2 is 30.

a. Find n

b. Find the first four terms of the expansion

2. Given that $f(x) \equiv x^3 - 10x + 12$

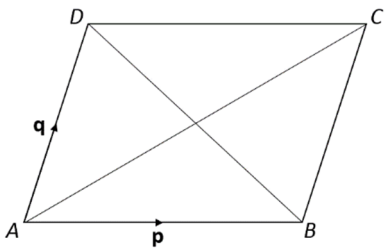
a. Show that $f(2) = 0$

b. Solve $f(x) = 0$

3. The points $(2, -3)$ and $(8, 7)$ are the ends of a diameter of a circle. Find the coordinates of the centre of the circle and the length of the diameter. What is the equation of the circle?

Section 2 – Consolidation of this week's topic. Please complete all questions.

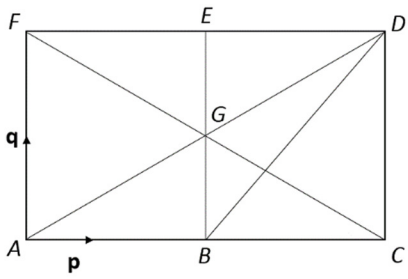
1. The diagram shows parallelogram $ABCD$, where $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$. Express these vectors in terms of **p** and **q**.



(a) $\overrightarrow{BC}$ (b) $\overrightarrow{DC}$ (c) $\overrightarrow{BA}$ (d) $\overrightarrow{CB}$ (e) $\overrightarrow{AC}$ (f) $\overrightarrow{BD}$ (g) $\overrightarrow{DB}$

(8 marks)

2. The diagram shows two squares $ABEF$ and $BCDE$, where $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AF} = \mathbf{q}$. Express these vectors in terms of **p** and **q**.



(a) $\overrightarrow{BD}$ (b) $\overrightarrow{AD}$ (c) $\overrightarrow{CF}$ (d) $\overrightarrow{AG}$

(9 marks)

3. Evaluate the magnitude, r, and the direction, θ, of these vectors, where θ is the anti-clockwise rotation from the positive *x*-direction and $-180° < \theta \leq 180°$

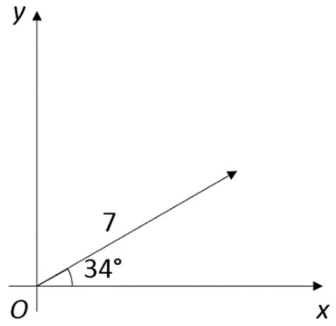
(a) $5\mathbf{i} + 2\mathbf{j}$ (b) $\begin{pmatrix}7\\9\end{pmatrix}$ (c) $-5\mathbf{j}$

(d) $\begin{pmatrix}-2\\3\end{pmatrix}$ (e) $3\mathbf{i} - 5\mathbf{j}$ (f) $\begin{pmatrix}-6\\-5\end{pmatrix}$

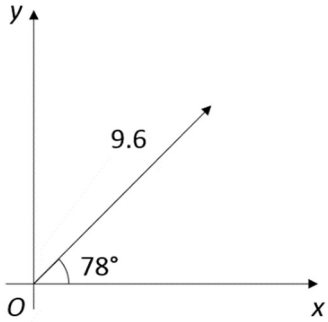
(12 marks)

4. Write these vectors in the form $x\mathbf{i} + y\mathbf{j}$ and $\begin{pmatrix} a \\ b \end{pmatrix}$

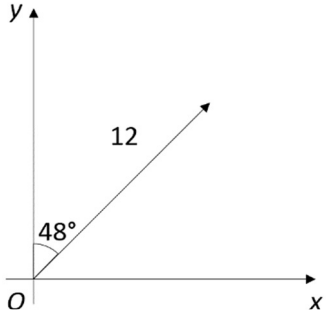
(a)



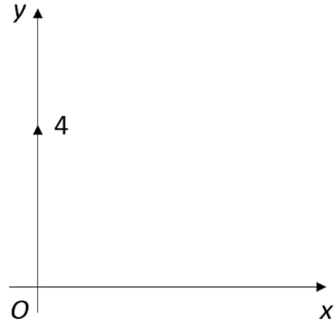
(b)



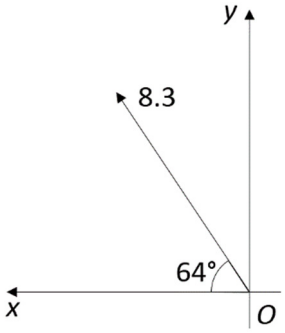
(c)



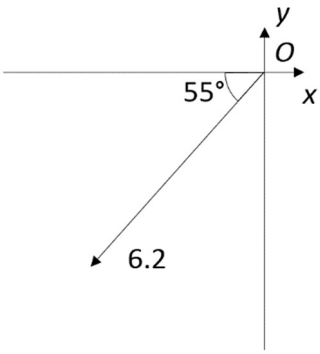
(d)



(e)



(f)



(18 marks)

5. Given vectors $\mathbf{p} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{q} = -2\mathbf{i} + 3\mathbf{j}$, and $\mathbf{r} = 4\mathbf{i} + \mathbf{j}$, calculate each of these vectors

(a) $\mathbf{p} + \mathbf{q}$ (b) $\mathbf{p} - \mathbf{r}$ (c) $2\mathbf{q} - \mathbf{p}$

(d) $2\mathbf{p} + 3\mathbf{r}$ (e) $|\mathbf{p}|$ (f) $|\mathbf{q} + \mathbf{r}|$

(13 marks)

6. Given vectors $\mathbf{p} = 3\mathbf{i} + u\mathbf{j}$, $\mathbf{q} = v\mathbf{i} - 4\mathbf{j}$, and $\mathbf{r} = 4\mathbf{i} - 6\mathbf{j}$, work out

(a) The values of u and v if $\mathbf{p} - \mathbf{q} = \mathbf{r}$

(b) The value of u if $\mathbf{p}$ and $\mathbf{r}$ are parallel

(4 marks)

7. Given $\mathbf{p} = -3\mathbf{i} + 4\mathbf{j}$, write down

(a) A vector parallel to $\mathbf{p}$ with magnitude 20

(b) The unit vector $\hat{\mathbf{p}}$ in the direction of $\mathbf{p}$

(4 marks)

(Total 68 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1. An aircraft has a speed in still air of 300 kmh^{-1}. A wind is blowing from the south at 80 kmh^{-1}. The pilot must fly to a point south-east of his present position.

    a. On what bearing should the pilot steer the aircraft?

    b. What is the resultant speed of the aircraft?

2. A ship which can travel at 12 kmh^{-1} in still water, is steered due north. A current of 9 kmh^{-1} from west to east pushes the ship off course.

    a. Find the ship's resultant velocity

    b. The ship is turned around with the intention of returning to its starting point. On what bearing should it be steered?