

Pure 15 – Applications of Differentiation

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

- 1 Find the equations of the following lines.
 - (i) parallel to $y = 4x - 1$ and passing through (2, 3)
 - (ii) perpendicular to $y = 2x + 7$ and passing through (1, 2)
 - (iii) parallel to $3y + x = 10$ and passing through (4, -1)
 - (iv) perpendicular to $3x + 4y = 12$ and passing through (-3, 0)
 - (v) parallel to $x + 5y + 8 = 0$ and passing through (-1, -6)

- 2 For each of these circles, find the coordinates of the centre and the radius.
 - (i) $x^2 + y^2 + 4x - 5 = 0$
 - (ii) $x^2 + y^2 - 6x + 10y + 20 = 0$
 - (iii) $x^2 + y^2 - 2x - 3y + 3 = 0$

- 3 The point C is (4, -2) and the point A is (6, 3).
Find the equation of the circle centre C and radius CA.

- 4 The line $2y + x = 10$ meets the circle $x^2 + y^2 = 65$ at P and Q.
Calculate the length of PQ.

- 5 The points P (-2, 6), Q (6, 0) and R (5, 7) all lie on a circle.
 - (i) Show that PR is perpendicular to QR.
 - (ii) Explain why the result from (i) shows that PQ is a diameter of the circle.
 - (iii) Hence calculate the equation of the circle.

Section 2 – Consolidation of this week’s topic. Please complete all questions.

1. For each of the following functions, find $\frac{d^2y}{dx^2}$.

(i) $y = x^3 - 3x^2 + 4x - 1$

(ii) $y = \frac{2}{x} - \frac{3}{x^2}$

(iii) $y = x^{\frac{3}{2}} + \sqrt{x}$ (6)

2. The displacement s metres of a particle at time t seconds is given by

$$s = t^2 + 3t - \frac{1}{t}.$$

Find the values of $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$ when $t = 2$ (4)

3. Find the equation of the tangent to the curve $y = x^4 - x + 1$ at the point with x -coordinate 1. (4)

4. Show that the equation of the normal to the curve $y = x^2 - x$ at the point (3, 6) is $x + 5y = 33$. Find the coordinates of the point where the normal meets the x -axis. (6)

5. Show that the tangent to the curve $y = x^3 + x + 2$ at the point P with x -coordinate 1 passes through the origin, and find the equation of the normal at this point. Given that the normal cuts the x -axis at the point Q, find the area of triangle OPQ. (10)

6. Find the equation of the tangent to the graph $y = \frac{1}{\sqrt{x}}$ at the point where $x = 1$. (5)

7. Find the equation of the normal to the graph $y = \frac{1}{x} - \frac{2}{x^2}$ at the point where $x = 2$. (5)

8. Find any stationary points on the following curves and determine their nature.

(i) $y = x - \frac{4}{x^2}$ (5)

(ii) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ (5)

Total: 50 Marks

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1. The curve $y = x^3 + px^2 + q$ has a minimum point at $(4, -11)$. Find the co-ordinates of the maximum point on the curve.
2. The curve $y = x^3 + ax^2 + bx + c$ passes through the point $(1, 1)$.
 - (i) Find an equation connecting a , b and c

The curve also has turning points when $x = -1$ and when $x = 3$

- (ii) Find two further equations connecting a , b and c .
Solve the three equations simultaneously to obtain values for a , b and c .