

Pure 16 – Integration

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. $f(x) \equiv 7 + 24x + 3x^2 - x^3$.
- a Find $f'(x)$. (2)
 - b Find the set of values of x for which $f(x)$ is increasing. (4)
2. $f(x) = (x + 1)(x - 2)^2$.
- a Sketch the curve $y = f(x)$, showing the coordinates of any points where the curve meets the coordinate axes. (3)
 - b Find $f'(x)$. (4)
 - c Show that the tangent to the curve $y = f(x)$ at the point where $x = 1$ has the equation $y = 5 - 3x$. (3)
3. a Find the coordinates of the stationary points on the curve $y = 2 + 9x + 3x^2 - x^3$. (6)
- b Determine whether each stationary point is a maximum or minimum point. (2)
 - c State the set of values of k for which the equation $2 + 9x + 3x^2 - x^3 = k$ has three solutions. (2)
4. $f(x) = 4x^3 + ax^2 - 12x + b$.
- Given that a and b are constants and that when $f(x)$ is divided by $(x + 1)$ there is a remainder of 15,
- a find the value of $(a + b)$. (2)
- Given also that when $f(x)$ is divided by $(x - 2)$ there is a remainder of 42,
- b find the values of a and b , (3)
 - c find the coordinates of the stationary points of the curve $y = f(x)$. (6)

Section 2 – Consolidation of this week’s topic. Please complete all questions.

1. For each of the following, find an expression for $f(x)$.

| | | |
|-------------------------------|----------------------------|-----|
| (i) $f'(x) = x^4$ | (ii) $f'(x) = 2x^7$ | |
| (iii) $f'(x) = \frac{1}{x^3}$ | (iv) $f'(x) = \sqrt[3]{x}$ | (4) |

2. The gradient function of a curve is given by $\frac{dy}{dx} = 4x^2 + x$.

- (i) Find the equation of the curve given that $y = 2$ when $x = 1$.
 (ii) Find the value of y when $x = 3$. (4)

3. Find an expression for y in terms of x if $\frac{dy}{dx} = (x - 1)(3x - 5)$ and $y = 2$ when $x = 1$. (5)

4. A curve has gradient function $\frac{dy}{dx} = \frac{x-3}{x^3}$ and passes through the point $(1, 1)$.
 Find the equation of the curve. (6)

5. Evaluate the following definite integrals.

| | | |
|--|-----------------------------------|----------------------------------|
| (i) $\int_{-1}^1 (4x+5)dx$ | (ii) $\int_{-1}^0 (6x^2 - 2x)dx$ | (iii) $\int_2^4 (x^2 - x + 3)dx$ |
| (iv) $\int_{-1}^2 (2 + x - x^2) dx$ | (v) $\int_{-1}^2 (x^3 - x + 4)dx$ | (vi) $\int_1^3 \frac{1}{x^3} dx$ |
| (vii) $\int_1^9 \frac{1}{\sqrt{x}} dx$ | | (21) |

(3 marks each)

6. Find the areas enclosed by the x axis and the following curves.

- (i) $y = (1 - x)(x + 2)$
 (ii) $y = 3x^2 - x^3$
 (iii) $y = x(x - 1)$
 (iv) $y = x^2 - 2x - 3$. (20)

(5 marks each)

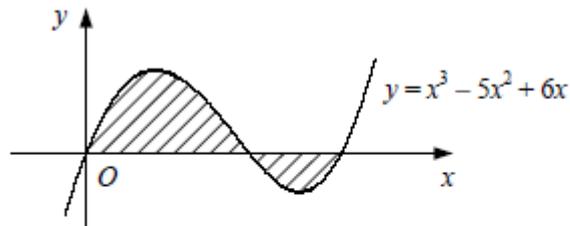
Total: 60 Marks

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1 Given that $\int_1^4 (3x^2 + ax - 5) dx = 18$, find the value of the constant a .

2 Given that $\int_{-1}^k (3x^2 - 12x + 9) dx = 16$, find the value of the non-zero constant k .

3



The diagram shows the curve with the equation $y = x^3 - 5x^2 + 6x$.

a Find the coordinates of the points where the curve crosses the x -axis.

b Show that the total area of the shaded regions enclosed by the curve and the x -axis is $3\frac{1}{12}$.