

Pure 22 - Logarithms

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

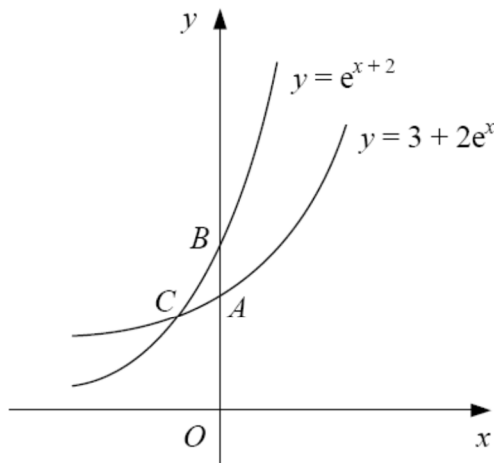


Figure 1

Figure 1 shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$ which cross the y -axis at the points A and B respectively.

(a) Find the exact length AB .

The two curves intersect at the point C .

(b) Find an expression for the x -coordinate of C and show that the y -coordinate of C is $\frac{3e^2}{e^2 - 2}$.

Section 2 – Consolidation of this week's topic. Please complete all questions.

1) Write as a single logarithm:

(a) $\log_2 7 + \log_2 3$

(b) $\log_2 36 - \log_2 4$

(c) $3\log_5 2 + \log_5 10$

(d) $2\log_6 8 - 4\log_6 3$

(e) $\log_{10} 5 + \log_{10} 6 - \log_{10} (\frac{1}{4})$

(8)

2) Write as a single logarithm, then simplify your answer:

(a) $\log_2 40 - \log_2 5$

(b) $\log_6 4 + \log_6 9$

(c) $2\log_{12} 3 + 4\log_{12} 2$

(d) $\log_8 25 + \log_8 10 - 3\log_8 5$

(e) $2\log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

(12)

3) Write in terms of $\log x$, $\log y$ and $\log z$:

(a) $\log(x^3y^4z)$ (b) $\log\left(\frac{x^5}{y^2}\right)$ (c) $\log(a^2x^2)$

(d) $\log\left(\frac{x\sqrt{y}}{z}\right)$ (e) $\log\sqrt{ax}$ (10)

4) Solve, giving your answer to 3 significant figures:

(a) $2^x=75$ (b) $\log_e x = 4.2$ (c) $\log_{10} x = 2.1$ (d) $4^{2x} = 100$
 (e) $9^{x+5} = 50$ (f) $7^{2x-1} = 23$ (g) $3^{x-1} = 9^{x+1}$ (h) $\log_{10}(2x+1) = -0.5$
 (i) $8^{3-x} = 2^x$ (j) $2^{4-3x} = 4^{x+5}$ (22)

5) Solve, giving your answer to 3 significant figures:

(a) $2^{2x} - 6(2^x) + 5 = 0$ (b) $3^{2x} - 15(3^x) + 44 = 0$
 (c) $7^{2x} + 12 = 7^{x+1}$ (d) $2\log_{10} x - \log_{10}(2x-1) = 1$
 (e) $3^{2x+1} - 26(3^x) - 9 = 0$ (f) $\log_e(x+1) + \log_e(x-1) = 1.2$ (18)

6) (a) Given that $\log_3 x = 2$, determine the value of x . (2)

(b) Calculate the value of y for which $2\log_3 y - \log_3(y+4) = 2$. (4)

(Total 76 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1. (a) Given that $3 + 2\log_2 x = \log_2 y$, show that $y = 8x^2$.

(b) Hence, or otherwise, find the roots α and β , where $\alpha < \beta$, of the equation

$$3 + 2\log_2 x = \log_2(14x-3).$$

(c) Show that $\log_2 \alpha = -2$.

(d) Calculate $\log_2 \beta$, giving your answer to 3 significant figures.

2. Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0$$