

Pure 25 – Differentiation From First Principles and Small Angle Approximations

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Find the unit vector in the direction $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.
2. The position vector of the point A is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ and $\overrightarrow{AB} = 3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and the coordinates of point C are $(1, -3, -2)$.

Find in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} ,

- a. the position vectors of B and C
- b. \overrightarrow{AC}

Find the exact value of

- c. The distance between A and C
 - d. $|\overrightarrow{OC}|$
3. Find the angles that the vector $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ makes with each of the positive coordinate axes to 1 d.p.
 4. Show that the function $f(x) = 4 - x(2x^2 + 3)$ is decreasing for all $x \in \mathbb{R}$.
 5. $f(x) = px^3 - 3px^2 + x^2 - 4$. When $x = 2$, $f''(x) = -1$. Find p .
 6. Given that $f(x) = x^2$ and $g(x) = 2x + 5$ solve $fg(x) = 9$.
 7. Find the inverse function of $f(x) = \frac{1}{x} - 3$, $x \in \mathbb{R}$, $2 < x < 5$.
 8. Find the equations of the tangents to the circle $x^2 + y^2 - 10x - 8y + 21 = 0$ at the points where the circle cuts the x axis.
 9. Solve the simultaneous equations:
 $\log(y - x) = 0$
 $2 \log y = \log(21 + x)$
 10. Solve $2^{2x} - 2^x = 6$.

Section 2 – Consolidation of this week’s topic. Please complete all questions.

1. a) When θ is small, show that the expression $\frac{5\cos 2\theta - \sin 3\theta - 4}{1 - \sin 5\theta}$ can be written as $2\theta + 1$. **(3 marks)**
b) Hence write down the value of $\frac{5\cos 2\theta - \sin 3\theta - 4}{1 - \sin 5\theta}$ when θ is small. **(1 mark)**
2. For small θ show that $\frac{\sin^2 3\theta}{1 - \cos 2\theta} \cong 4.5$. **(3 marks)**
3. Solve $\frac{\sin^2 5\theta + 2\theta}{\tan \theta} = 3$ for the case when θ can be assumed to be small. **(3 marks)**
4. Differentiate the following from first principles:
 - a. $\sin x$ **(5 marks)**
 - b. $\cos 3x$ **(7 marks)**
 - c. $4\cos x + 3x^2$ **(5 marks)**

Total: 27 marks