

Pure 27 – Differentiation: Chain Rule, Product & Quotient Rules

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.

Please complete all questions.

1. Find the equation of the normal to the curve $y = x^2 - \frac{8}{\sqrt{x}}$ at the point (4,12).
2. Find the values of x for which $f(x) = 3x^2 + 8x + 2$ is an increasing function.
3. Given that $y = 3x^5 + \frac{4}{x^2}$, find $\frac{d^2y}{dx^2}$.
4. $f(x) = px^3 - 3px^2 + x^2 - 4$. When $x = 2$, $f''(x) = -1$. Find the value of p .
5. For the function $f(x) = 2x^3 - 15x^2 + 24x + 6$, find:
 - a) The coordinates of the stationary points
 - b) The nature of these stationary points (ie whether they are minimum or maximum points)
6. Given that $x \neq 0$ find the set of values for which $5 > \frac{4}{x}$. Give your answer using set notation.
7. The equation $kx^2 - 2kx + 3 = 0$ where k is a constant has no real roots. Prove that k satisfies the inequality $0 \leq k < 3$.
8. A straight line passes through the points $(a, 4)$ and $(3a, 3)$. Its equation is $x + 6y + c = 0$. Find the values of a and c .
9. A line is perpendicular to the line $3x + 8y - 11 = 0$ and passes through the point $(0, -8)$. What is the equation of the line in the form $ay + bx + c = 0$?
10. A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P .

Section 2 – Consolidation of this week’s topic.
Please complete all questions.

- 1)** Differentiate with respect to x and simplify where possible:
 a) $y = e^{x^2}$ b) $f(x) = \sin^2 x$ c) $y = e^{2 \sin x}$
 d) $f(x) = \sin(\sqrt{x}) + \sqrt{\sin x}$ e) $y = 2 \ln(3x + 5)$ **[6]**
- 2)** Differentiate with respect to x and simplify where possible:
 a) $y = 5x(3x^2 + 1)^3$ b) $f(x) = \sin 4x \cos 5x$ c) $y = \ln 2x \sin 3x$ **[7]**
- 3)** Differentiate with respect to x and simplify where possible:
 a) $y = \frac{x}{x^2 - 1}$ b) $f(x) = \frac{e^x}{\cos x}$ c) $y = \frac{\sqrt{x}}{\sqrt{x} + 1}$ **[5]**
- 4)** Differentiate with respect to x and simplify where possible:
 a) $y = x^2 e^{-3x}$ b) $f(x) = \sqrt{x} \ln 3x$ c) $y = x \ln \left(\frac{x-1}{x+1} \right)$
 d) $f(x) = \ln(\cos x)$ **[11]**
- 5)** A curve has the equation $y = xe^{x^2}$. **[8]**
 a) Find the equation of the tangent to the curve at $x = 1$.
 b) Find the coordinates of the points where the tangent cuts the x and y axes.
 c) Find the area of the triangle bounded by this tangent and the x and y axes.
- 6)** The curve $y = (x + 3)(x - 1)^3$ crosses the x axis at 2 points P and Q and has a turning point at R . Given that the x coordinate of P is less than the x coordinate of Q , write down the coordinates of P and Q and find the coordinates of R , stating whether it is a minimum or maximum. **[11]**
- 7)** $f(x) = x\sqrt{2x + 12}$, $x \geq -6$. **[13]**
 a) Find $f'(x)$
 b) Show that $f''(x) = \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$.
 c) Find the turning point of the curve and determine its nature.

Total: 61 Marks