Statistics 14 – Hypothesis Testing 2 – Solutions

Section 1

1a)

x	1	2	3	4
$P(X=x)$	k	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{4}$

$k+\frac{k}{2}+\frac{k}{3}+\frac{k}{4}=1 \Rightarrow \frac{25k}{12}=1 \Rightarrow k=\frac{12}{25}$

b)

x	1	2	3	4
$P(X=x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

c) i) $P(X>2) = \frac{4}{25}+\frac{3}{25}=\frac{7}{25}$

ii) $P(1<X<4) = \frac{6}{25}+\frac{4}{25}=\frac{10}{25}=\frac{2}{5}$

iii) $P(X>4)=0$

2. a)

	S	S'	
T			0.4
T'	0.18		0.6
	0.3	0.7	

$P(S \cap T) = 0.3-0.18=0.12$

$P(S)\times P(T) = 0.3\times 0.4 = 0.12$

$\Rightarrow P(S)\times P(T) = P(S\cap T)$ → S, T are independent

b) i) $P(S \cup T) = 1-P(S'\cap T') = 1-(0.6-0.18) = 1-0.42 = 0.58$

ii) $P(S'\cup T) = 1-P(S\cap T') = 1-0.18 = 0.82$

3.

x	1	2	3	4
$P(X=x)$	k	k	$3k$	$3k$

a) $k+k+3k+3k=1 \Rightarrow 8k=1 \Rightarrow k=\frac{1}{8}$

b) $P(X>1) = k+3k+3k = 7k = 7\left(\frac{1}{8}\right) = \frac{7}{8}$

Section 2

1. a) The critical value is the boundary of the critical region.

If H_1 is $p>\ldots$ critical value is smallest value in critical region

If H_1 is $p<\ldots$ " " " largest " " " "

b) Critical region is range of values in which H_1 is accepted

c) Acceptance region is " " " " H_0 " "

(3)

2. a) $H_0: p=0.2$

$H_1: p>0.2$

$X \sim B(10, 0.2)$

95% → H_0 | H_1 ← 5%, boundary c

$P(X \geq c) = 0.05$

$\Rightarrow 1-P(X \leq c-1) = 0.05$

$\Rightarrow P(X \leq c-1) = 0.95$

From calc $P(X \leq 3) = 0.8791$, $P(X \leq 4) = 0.9672$ } $c-1$ between 3 and 4

$\Rightarrow c$ between 4 and 5

ie Critical region is $x \geq 5$

b) $x=4 \Rightarrow$ Accept H_0, Reject H_1

(3)

3. $H_0: p=0.15$

$H_1: p<0.15$

$X \sim B(20, 0.15)$

5% → H_1 | H_0 ← 95%, boundary c

$P(X \leq c) = 0.05$

$P(X \leq 0) = 0.0387$, $P(X \leq 1) = 0.1755$ } c between 0 and 1

ie Critical region is $x \leq 0$ (ie $x=0$)

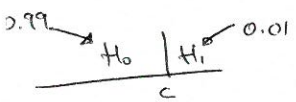
$\Rightarrow$ Acceptance region $x \geq 1$

(3)

4. X = no. spaces available

$X \sim Bin(10, 0.35)$ ← 2 college weeks = 2×5 days

$H_0: p = 0.35$
$H_1: p > 0.35$ ✓



$P(X \geq c) = 0.01$
$\Rightarrow P(X \leq c-1) = 0.99$

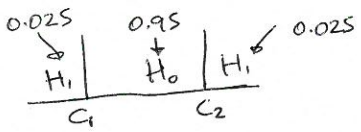
From calc $P(X \leq 6) = 0.9739$, $P(X \leq 7) = 0.9951$ } $c-1$ between 6 and 7 $\Rightarrow c$ between 7 and 8 ✓

i.e Critical region is $x \geq 8$ ✓ (3)

5. a) X = no. A grades

$X \sim Bin(20, 0.4)$

$H_0: p = 0.4$
$H_1: p \neq 0.4$ ✓



$P(X \leq C_1) = 0.025$

$P(X \leq 3) = 0.0159$, $P(X \leq 4) = 0.0509$ } C_1 between 3 and 4

$P(X \geq C_2) = 0.025$

$\Rightarrow P(X \leq C_2 - 1) = 0.975$ ✓

$P(X \leq 11) = 0.9434$, $P(X \leq 12) = 0.9789$ } $C_2 - 1$ between 11 and 12 $\rightarrow C_2$ between 12 and 13

i.e Critical regions are $x \leq 3$ and $x \geq 13$ ✓

b) $x = 11$ in acceptance region $\Rightarrow$ Accept H_0, Reject H_1 ✓

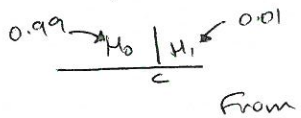
i.e. insufficient evidence that proportion of students achieving A's has changed

c) 1% < 5% $\Rightarrow$ If reject H_1 at 5% would also reject H_1 at 1%
$\Rightarrow$ will not be different ✓

d) Expect $np = 20 \times 0.4 = 8$. Therefore would clearly be in acceptance region if $x = 8$. ✓ (6)

6 a) X = no. students late $X \sim Bin(20, 0.1)$

$H_0: p = 0.1$
$H_1: p > 0.1$ ✓



$P(X \geq c) = 0.01$
$\Rightarrow P(X \leq c-1) = 0.99$ ✓

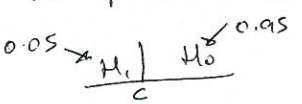
From calc $P(X \leq 5) = 0.9883$, $P(X \leq 6) = 0.9976$ } $c-1$ between 5 and 6 $\Rightarrow c$ between 6 and 7

i.e critical region is $x \geq 7$ $\Rightarrow$ 7 is smallest number ✓

b) X unlikely to follow a binomial distribution as not independent (one person being late makes it more likely others will be late) ✓ (4)

7. X = no. seeds germinating $X \sim Bin(10, 0.85)$

$H_0: p = 0.85$
$H_1: p < 0.85$ ✓



$P(X \leq c) = 0.05$

From calc $P(X \leq 6) = 0.0499$, $P(X \leq 7) = 0.1798$ } c between 6 and 7 ✓

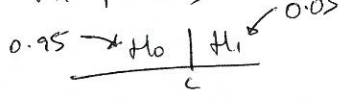
i.e critical region is $x \leq 6$ ✓ (3)

Total = (25)

Section 3

X = no. favour red $X \sim Bin(6, 0.3)$

$H_0: p = 0.3$
$H_1: p > 0.3$



$P(X \geq c) = 0.05$
$P(X \leq c-1) = 0.95$
$P(X \leq 3) = 0.9295$, $P(X \leq 4) = 0.989$ } $c-1$ between 3 and 4 $\Rightarrow c$ between 4 and 5

$\therefore$ critical region is $x \geq 5$