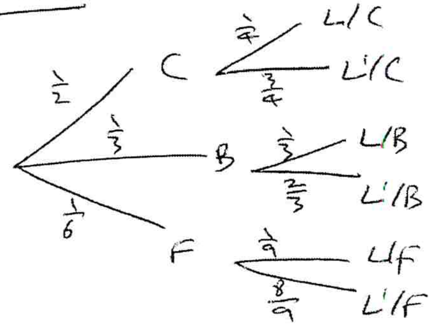
Statistics 18 - Normal Distribution - Solutions

Section 1

1(a)



b) i) $P(F \cap L) = \frac{1}{6} \times \frac{1}{9} = \frac{1}{54}$

ii) $P(L') = P(C \cap L') + P(B \cap L') + P(F \cap L')$

$= \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{6} \times \frac{8}{9}$

$= \frac{3}{8} + \frac{2}{9} + \frac{4}{27} = \frac{161}{216} = 0.7454$

iii) $P(F'/L) = \dfrac{P(C \cap L) + P(B \cap L)}{P(L)} = \dfrac{\frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{3}}{1 - 0.7454}$

$= \dfrac{\frac{17}{72}}{\frac{55}{216}} = \frac{51}{55} = 0.927$

2.

X	1	2	3	4	5
P(X)	k	4k	9k	4k	5k

a) $k + 4k + 9k + 4k + 5k = 1 \Rightarrow 23k = 1 \Rightarrow k = \frac{1}{23}$

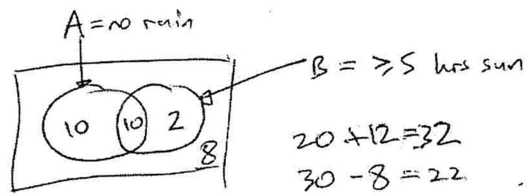
b) $P(X > 2.4) = P(X = 3, 4, 5) = 9k + 4k + 5k = 18k = \frac{18}{23}$

c) $Y =$ no. times $X > 2.4$ $\quad Y \sim Bin(15, \frac{18}{23})$

$P(Y \geq 13) = 1 - P(Y \leq 12) = 1 - 0.6643 = 0.3357$

3.

a)



$20 + 12 = 32$

$30 - 8 = 22$

$32 - 22 = 10$ ← 10 in overlap

b) i) $P(A') = 1 - \frac{20}{30} = \frac{1}{3}$

ii) $P(A \cup B') = \frac{10 + 10 + 8}{30} = \frac{14}{15}$

iii) $P(A'/B) = \frac{2}{12} = \frac{1}{6}$

c) $P(A \cup B')$ means either no rain, or less than 5 hours of sun (or both)

$P(A'/B)$ means Given it had at least 5 hours of sun, find the probability it rained

d) $X =$ no. days rained $\quad X \sim Bin(30, \frac{1}{3})$

$P(X = 11) = {}^{30}C_{11} \left(\frac{1}{3}\right)^{11} \left(\frac{2}{3}\right)^{19} = 0.1391$

e) Assumed probability of rain in 1997 and 1987 is the same

Also assumed it is independent.

Both of these assumptions are very unlikely to be true - not reasonable

4 a) If B has no effect on P(A) then $P(A/B) = P(A)$

Alternatively $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$

b) If A is not affected by B happening, A is also unaffected by B not happening

$P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)(1 - P(B)) = P(A)P(B')$

c) $P(A' \cap B) = P(B) - P(A \cap B) = P(B) - P(A)P(B) = P(B)(1-P(A)) = P(B)\,P(A')$

Alternatively if A has no effect on B, then A not happening can have no effect on B either.

d) If A and B were mutually exclusive A happening would make it impossible for B to happen, i.e not independent. Since A and B are independent they can not be mutually exclusive

e) If A and B are exhaustive, A not happening would make it certain for B to happen, i.e not independent. Since A and B are independent they can not be exhaustive.

Section 2

1. $X \sim N(20, 3^2)$

a) 95% between 2 s.devs i.e between $20-2(3)$ and $20+2(3)$
$\Rightarrow$ between 14 and 26

b) inflection at $\mu \pm \sigma$ i.e 23 and 17

2

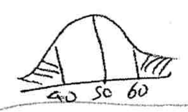
2. $X \sim N(50, 6^2)$

a) i) $P(X < 60) = P(-100 < X < 60) = 0.9522$

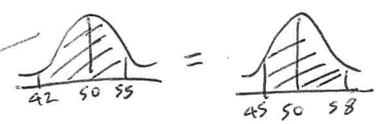
ii) $P(X > 40) = P(40 < X < 1000) = 0.9522$

iii) $P(42 < X < 55) = 0.7065$

iv) $P(45 < X < 58) = 0.7065$

b) i) = ii) due to symmetry of graph about 50 

iii) = iv) " " " " " "



6

3. a) $X \sim N(1017, 8^2)$ $\quad \sigma^2 = 64 \Rightarrow \sigma = 8$

b) i) $P(X > 1025) = P(1025 < X < 2000) = 0.1587$

ii) $P(X < 1000) = P(0 < X < 1000) = 0.0168$

iii) $1 - P(990 < X < 1000) = 1 - 0.0164 = 0.9836$

4

4. X = time (mins)

$X \sim N(12\tfrac{1}{6}, (1\tfrac{1}{3})^2)$

a) $P(X < 11) = P(0 < X < 11) = 0.1908$.

34 runners $\Rightarrow$ expect $34 \times 0.1908 = 6.49$

i.e 6 runners

b) Assumed times are roughly symmetric, with 95% of data within 2 standard deviations of the mean.

5

5. $X \sim N(15, 2^2)$

a) $P(X > 18) = P(18 < X < 100) = 0.0668$

b) Y = no. values above 18

$Y \sim Bin(10, 0.0668)$

i) $P(Y = 2) = 0.1155$

ii) $P(Y < 2) = P(Y \leq 1) = 0.8594$

5

6. X = weight of coffee Y = weight of glass

$X \sim N(101.5,\ 7.5^2)$ $Y \sim N(269,\ 5.5^2)$

$P(X < 97)$

$= P(0 < X < 97) =$

$= 0.2743$

$P(Y < 259)$

$= P(0 < Y < 259)$

$= 0.0345$

$P(X < 97 \cap Y < 259) = 0.2743 \times 0.0345 = 0.00945$

3

7. $X \sim N(15, 3^2)$

a) i) $P(X < 11.5) = P(-100 < X < 11.5) = 0.1217$

ii) $P(X > 12.5) = P(12.5 < X < 100) = 0.7977$

iii) $P(11.5 < X < 12.5) = 0.0807$

b) Would expect $P(X < 11.5) + P(X > 12.5) + P(11.5 < X < 12.5) = 1$

In this case they add to 1.0001 due to rounding

c) Because X is continuous $P(X = 12) \neq P(11.5 < X < 12.5)$

For all continuous data the probability that it is an exact value is 0 i.e. $P(X = 12) = 0$.

Charles is incorrect.

5

Total = 30