Statistics 19 – Inverse Normal Distribution – Solutions

Section 1

1\. a) $P(X < Q_1) = \frac{1}{4}$

b) $P(X > Q_2) = \frac{1}{2}$

c) i) Y = no. values below Q_3

$Y \sim Bin(4, \frac{3}{4})$

$P(Y = 4) = 0.3164$

ii) Z = no. values between Q_1 and Q_2

$Z \sim Bin(4, \frac{1}{4})$

$P(Z \geq 2) = 1 - P(Z \leq 1) = 1 - 0.7383 = 0.2617$

2\. $X \sim N(70, 8^2)$

a) $P(X < 60) = P(0 < X < 60) = 0.1056$

b) $P(X > 80) = P(80 < X < 1000) = 0.1056$

c) $P(X = 55) = 0$

d) $P(X < 62 \cup X > 78) = 1 - P(62 < X < 78) = 1 - 0.6827 = 0.3173$

3\. a) (X is continuous data)

Median & Mean are very close

$n = 30 + 31 = 61$

Normal $\Rightarrow$ 68% of data within one s.dev of mean

$\mu = 1018 \quad \sigma \approx 10 \quad \Rightarrow \quad P(1008 < X < 1028) = 0.68$

... 68% data within on s.dev of mean

b) $X \sim N(1018, 10.027^2)$

$P(1015 < X < 1025) = 0.3751$

4\. X = no. phones with cracked screen

$X \sim Bin(\cancel{4}0, 0.1)$

$H_0: p = 0.1$

$H_1: p < 0.1$

10% H_1 | H_0 90%

c

$P(X \leq 0) = 0.0147$

$P(X \leq 1) = 0.0804$

$P(X \leq 2) = 0.2228$

$\Big\}$ c between 1 and 2

Initially $n < c$

Then $n+1 > c$

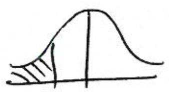
$\Big\} \Rightarrow n = 1$

5\. $X \sim N(\mu, \sigma^2)$

a) i) 

μ a

$P(X < a) > 0.5 \Rightarrow a > \mu$

ii) 

a μ

$P(X < a) < 0.5 \Rightarrow a < \mu$

iii) 

a μ

$P(X > a) > 0.5 \Rightarrow a < \mu$

iv) 

μ a

$P(X > a) < 0.5 \Rightarrow a > \mu$

b) $a = \mu \Rightarrow P(X < a) = P(X > a) = 0.5$

Section 2

1\.a) Continuous data

Data roughly symmetrical with modal group in middle

b) $X \sim N(47.5, 10^2)$

i) $P(35 < X < 45) = 0.2956$

$n = 100 \Rightarrow$ Predict $0.2956 \times 100 = 29.56 \approx 30$ plums

ii) $P(m < 25) = P(-100 < m < 25) = 0.0122$

$n = 100 \Rightarrow$ Predict $0.0122 \times 100 = 1.22 \approx 1$ plum

c) $35 \le m < 45$ freq $= 29$ model $= 30$

$m < 25$ freq $= 0$ model $= 1$

$\}$ Both answers very close to reality $\Rightarrow$ model is suitable

d) This is not advisable. Plum crops likely to vary from year to year. (will probably still be normally distributed)

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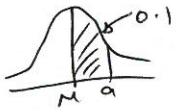
2\. $X \sim N(40, 10^2)$

a) $a = 31.58$

b) $P(X > a) = 0.7 \Rightarrow P(X < a) = 0.3 \Rightarrow a = 34.76$

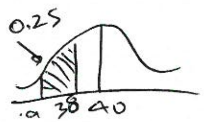
c)



$P(X < M) = 0.5$

$\Rightarrow P(X < a) = 0.6 \Rightarrow a = 42.53$

d)



$P(X < 38) = P(-100 < X < 38) = 0.4207$

$P(a < X < 38) = 0.25 \Rightarrow P(X < a) = 0.4207 - 0.25 = 0.1707$

$P(X < a) = 0.1707 \Rightarrow a = 30.49$

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3\. $X \sim N(50, 25) \Rightarrow X \sim N(50, 5^2)$

a) Median $\approx$ Mean $= 50$

b) Mode $=$ Mean $= 50$

c) $P(X < LQ) = 0.25 \Rightarrow LQ = 46.63$

d) $P(X < UQ) = 0.75 \Rightarrow UQ = 53.37$

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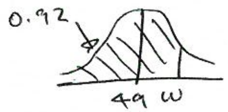
4\. X = weight (kg)

$X \sim N(49, 3^2)$

a) $P(X > 55) = P(55 < X < 100) = 0.0228$

b) $P(X > w) = 0.92 \Rightarrow P(X < w) = 0.08 \Rightarrow w = 44.78$



c) Y = no. bags that weigh above 55 kg

$Y \sim Bin(5, 0.0228)$

i) $P(Y = 1) = 0.1040$

ii) $P(Y = 0) = 0.8911$

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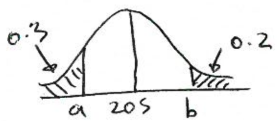
5\. X = weight orange (g)

$X \sim N(205, 25^2)$

a i) $P(X < 250) = P(-100 < X < 250) = 0.9641$

ii) $P(200 < X < 250) = 0.5433$

b)



i) $P(X < a) = 0.3 \Rightarrow a = 191.9$ g

ii) $P(X > b) = 0.2$

$\Rightarrow P(X < b) = 0.8 \Rightarrow b = 226.0$ g

c) p not fixed (without replacement)

[illegible] likely to try to sell smaller oranges

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