Statistics 14 – Hypothesis Testing (2)

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please <u>complete</u> all questions.

1. A biased four-sided dice with faces numbered 1-4 is rolled. The number on the bottom most face is modelled as a random variable X.

   Given that Given that $P(X = x) = \frac{k}{x}$

   (a) Find the value of k
   (b) Give the probability distribution of X in table form
   (c) Find the probability that:
      i. $X > 2$ ii $1 < X < 4$ iii $X > 4$

2. S and T are two events such that *P(S)* = 0.3, *P(T)* = 0.4 and *P(S but not T)* = 0.18
   (a) Show that S and T are independent.
   (b) Find:
      i.   *P(S ∪ T)*   ii.   *P(S' ∪ T)*

3. The random variable has the probability function

$$P(X = x) \begin{cases} kx & x = 1,3 \\ k(x - 1) & x = 2,4 \end{cases}$$

   where *k* is a constant

      (a) Find the value of *k*.
      (b) Find $P(X > 1)$

Section 2 – Consolidation of this week's topic. Please <u>complete</u> all questions.

1. Explain what you mean by the following;
   (a) critical value   (b) critical region   (c) acceptance region.   **(3 marks)**

2. $H_0: p = 0.20; \ H_1: p > 0.20; \ n = 10$
   a) Find the critical region for the hypothesis test at the 5% level of significance. **(2 marks)**
   b) Hence conclude the test if the test statistic is 4.   **(1 mark)**

3. $H_0: p = 0.15; \ H_1: p < 0.15; \ n = 20$. Find the critical region and acceptance region using a 5% level of significance   **(3 marks)**

4. The probability that a car parking space is available at a college is known to be 0.35. After a campaign in tutorials encouraging students to car share, Charles thinks it will be more likely that he will find a space. He decides to monitor the situation over 2 college weeks. If he uses a 1% level of significance determine the critical region(s)   **(3 marks)**

5. In the old A level specification 40% of students at a college would achieve an A grade. A teacher thinks this might change with the new specification and so carries out a hypothesis test with a 5% significance level on her class of 20 students.
   a) Determine the critical region(s) **(3 marks)**
   b) If 11 students achieve an A grade, does this provide sufficient evidence at the 5% level that the proportion of students achieving an A grade has changed? **(1 mark)**
   c) Is it possible the answer to b) would be different if the 1% significance level had been used? **(1 mark)**
   d) If only 8 student had achieved an A grade, why would it be inefficient to carry out a hypothesis test regardless of the significance level used? **(1 mark)**

6. 1 in 10 students are usually late to an 8.45 Tuesday lesson. A teacher is stuck in traffic on his way to work the following Tuesday and so expects more students to be late than usual. To keep himself entertained he decides to carry out a hypothesis test.
   a) Given that there are 20 students in his class, and he chooses to use a 1% level of significance, determine the smallest number of students that will need to be late in order for the teacher's expectations to be supported. **(3 marks)**
   b) Why is the use of a hypothesis test in this scenario dubious? **(1 mark)**

7. A seed merchant maintains their stock in carefully monitored conditions. At the end of one year she found that the monitoring system had broken down and there was a danger that the seed might have been damaged by frost. She decided to check a sample of 10 seeds to see if the proportion p that germinated had been reduced from the usual 0.85. Find the critical region for a 1-tailed test using a 5% significance level. **(3 marks)**

(Total 25 marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

'Research shows 3 out of every 10 children say their favourite colour is red' A teacher believes the proportion should have been higher. She then asked 6 children at random what their favourite colour was. Find how many children must choose red for this to be a significant result, using a 5% significance level.