

## Statistics 18 – Normal Distribution

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics.

Please complete all questions.

1. On a randomly chosen day, the probability that Michelle travels to school by car, by bicycle or on foot is  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{6}$  respectively. The probability of being late when using these methods of travel is  $\frac{1}{4}$ ,  $\frac{1}{3}$  and  $\frac{1}{9}$  respectively.
  - a) Draw a tree diagram to represent this information.
  - b) Find the probability that on a randomly chosen day:
    - i. Michelle travels by foot and is late
    - ii. Michelle is not late
    - iii. Given Michelle is late, find the probability that she did not travel on foot

2. A discrete random variable is given by the equation

$$P(X = x) = \begin{cases} kx^2 & x = 1,2,3 \\ kx & x = 4,5 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find the value of  $k$
  - b) Find the probability that  $X$  is greater than 2.4
  - c) 15 independent values of  $X$  are taken. Find the probability that  $X$  is greater than 2.4 for at least thirteen of them
3. In Heathrow there were 20 days with no rain during the June of 1987. There were 12 days with at least 5 hours of sunshine. There are 8 days where it rained and there were less than 5 hours of sunshine. Let  $A$  and  $B$  represent “no rain” and “at least 5 hours of sunshine” respectively.
    - a) Draw a Venn diagram
    - b) Find
      - i.  $P(A')$
      - ii.  $P(A \cup B')$
      - iii.  $P(A'/B)$
    - c) Interpret the questions in bii) and biii)  
(For example bi) means the probability that there was some rain)
    - d) Abdul uses the data from 1987 to make predictions for the weather in 1997. Find the probability that it rained on exactly 11 days in June 1997 in Heathrow.
    - e) What assumptions have you made in answering d)? Are these assumptions sensible?

4. A and B are two independent events. Explain why:

- a)  $P(A/B) = P(A)$
- b)  $P(A/B') = P(A)$
- c) A' and B are independent
- d) A and B are not mutually exclusive
- e) A and B are not exhaustive

## Section 2 – Consolidation of this week's topic.

Please complete all questions.

1.  $X$  follows a normal distribution with mean 20 and standard deviation 3.

- a) Between what values would you expect 95% of the data to lie? (1 mark)
- b) What are the points of inflections on the graph of  $X$ ? (1 mark)

2.  $X \sim N(50, 6^2)$

a) Use your calculator to find:

- i.  $P(X < 60)$  (1 mark)
- ii.  $P(X > 40)$  (1 mark)
- iii.  $P(42 < X < 55)$  (1 mark)
- iv.  $P(45 < X < 58)$  (1 mark)

b) Why does it make sense that some of the answers in a) are the same (2 marks)

3.  $X$  = The Daily mean pressure for Hurn in 2015.

Charles uses the large data set to calculate that the mean of  $X$  is 1017 (hPa) and the variance is 64 (hPa<sup>2</sup>), and decides to model  $X$  with the normal distribution.

- a) State the distribution of  $X$  (1 mark)
- b) Find the probability that during 2015 a random day in Hurn has a daily mean pressure that is:
  - i. Above 1025 (1 mark)
  - ii. Below 1000 (1 mark)
  - iii. Not between 990 and 1000 (1 mark)

4. A school's cross country event is known to be completed in a mean time of 12 minutes 10 seconds, with a standard deviation of 1 minute 20 seconds. One year 34 runners took part, and a commendation given to any runner that completed the course in less than 11 minutes.

- a) State the distribution of the time of the runners (1 mark)
- b) Estimate the number of runners receiving the commendation (2 marks)
- c) State any distributional assumptions made (2 marks)

5.  $X \sim N(15, 2^2)$
- a) Find  $P(X > 18)$  (1 mark)
- b) 10 independent values of  $X$  are taken. Find the probability that:
- i. Exactly 2 are above 18 (2 marks)
  - ii. Less than 2 are above 18 (2 marks)
6. The weight of coffee in glass jars labelled 100g is normally distributed with mean 101.5g and standard deviation 7.5g. The weight of an empty glass is normally distributed with mean 269g and standard deviation 5.5g. The weight of a glass jar is independent of the weight of the coffee it contains. Find the probability that a randomly selected jar weighs less than 259g and contains less than 97g of coffee. (3 marks)
7.  $X \sim N(15, 3^2)$
- a) Use your calculator to find to four decimal places:
- i.  $P(X < 11.5)$  (1 mark)
  - ii.  $P(X > 12.5)$  (1 mark)
  - iii.  $P(11.5 < X < 12.5)$  (1 mark)
- b) What would you expect to find about your answers to a)? (1 mark)
- c) Charles claims that the answer to aiii) is the same as  $P(X = 12)$ . Is Charles correct? Give reasons. (1 mark)

**Total: 30 Marks**